MTH 1125 Test #1 - (12 pm class) - Solutions

Fall 2017

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 2}\frac{x^2+2x+12}{x^2+6x-12} =$

Step #1 Try Plugging In:

$\lim_{x\to 2}\frac{x^2+2x+12}{x^2+6x-12} = \frac{(2)^2+2(2)+12}{(2)^2+6(2)-12} = 5$

i.e., $\lim_{x\to 2}\frac{x^2+2x+12}{x^2+6x-12} = 5$

2. Compute: $\lim_{x\to -1}\frac{x^2-5x-6}{x^2-2x-3} =$

(a) $\lim_{x\to -1}\frac{x^2-5x-6}{x^2-2x-3} = \frac{(-1)^2-5(-1)-6}{(-1)^2-2(-1)-3} = \frac{0}{0}$ No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

$\lim_{x\to -1}\frac{x^2-5x-6}{x^2-2x-3} = \lim_{x\to -1}\frac{(x+1)(x-6)}{(x+1)(x-3)} = \lim_{x\to -1}\frac{(x-6)}{(x-3)} = \frac{(-1)-6}{(-1)-3} = \frac{-7}{-4} = \frac{7}{4}$

i.e., $\lim_{x\to -1}\frac{x^2-5x-6}{x^2-2x-3} = \frac{7}{4}$

3. Compute: $\lim_{x\to\ 2} \frac{x^2-2x-15}{x^2+2x-8} =$

Step #1 Try Plugging in:

$$\lim_{x\to\ 2} \frac{x^2-2x-15}{x^2+2x-8} = \frac{(2)^2-2(2)-15}{(2)^2+2(2)-8} = \frac{-15}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

No Good! "Factoring and Cancelling" only works when Step #1 yields $\frac{0}{0}$.

Step #3 Analyze the one-sided limits:

$$\lim_{x\to\ 2^-} \frac{x^2-2x-15}{x^2+2x-8} = \lim_{x\to\ 2^-} \frac{x^2-2x-15}{(x+4)(x-2)} = \frac{-15}{(6)(-\varepsilon)} = \frac{\left(-\frac{15}{6}\right)}{(-\varepsilon)} = +\infty$$

$$\begin{array}{ll} & x \to 2^- \\ \Rightarrow & x < 2 \\ \Rightarrow & x - 2 < 0 \end{array}$$

$$\lim_{x\to\ 2^+} \frac{x^2-2x-15}{x^2+2x-8} = \lim_{x\to\ 2^+} \frac{x^2-2x-15}{(x+4)(x-2)} = \frac{-15}{(6)(+\varepsilon)} = \frac{\left(-\frac{15}{6}\right)}{(+\varepsilon)} = -\infty$$

$$\begin{array}{ll} & x \to 2^+ \\ \Rightarrow & x > 2 \\ \Rightarrow & x - 2 > 0 \end{array}$$

Since the one-sided limits are not equal, $\lim_{x\to\ 2} \frac{x^2-2x-15}{x^2+2x-8}$ **Does Not Exist!**

4. Compute: $\lim_{x\to -\infty} \frac{x^4+4x^3-8x}{9x^3+4x-5} =$

$$\lim_{x\to\ -\infty} \frac{x^4+4x^3-8x}{9x^3+4x-5} = \lim_{x\to\ -\infty} \frac{x^4}{9x^3} = \lim_{x\to\ -\infty} \frac{x}{9} = -\infty$$

i.e., $\lim_{x\to\ -\infty} \frac{x^4+4x^3-8x}{9x^3+4x-5} = -\infty$

5. $f(x) = \frac{x^2-4x+3}{x^2-3x-10}$ Find the asymptotes and graph

Verticals

1. Find x-values that cause division by zero.

$\Rightarrow x^2 - 3x - 10 = 0$

$\Rightarrow (x+2)(x-5) = 0$

$\Rightarrow x = -2$ and $x = 5$ are possible vertical asymptotes.

2. Compute the one-sided limits.

$$\lim_{x\to -2^-} \frac{x^2-4x+3}{x^2-3x-10} = \lim_{x\to -2^-} \frac{x^2-4x+3}{(x+2)(x-5)} = \frac{15}{(-\varepsilon)(-7)} = \frac{15}{(\varepsilon)(7)} = \frac{\left(\frac{15}{7}\right)}{\varepsilon} = +\infty$$

$$\begin{array}{ll} & x \to -2^- \\ \Rightarrow & x < -2 \\ \Rightarrow & x+2 < 0 \end{array}$$

$$\lim_{x\to -2^+} \frac{x^2-4x+3}{x^2-3x-10} = \lim_{x\to -2^+} \frac{x^2-4x+3}{(x+2)(x-5)} = \frac{15}{(\varepsilon)(-7)} = \frac{\left(-\frac{15}{7}\right)}{\varepsilon} = -\infty$$

$$\begin{array}{ll} & x \to -2^+ \\ \Rightarrow & x > -2 \\ \Rightarrow & x+2 > 0 \end{array}$$

Since the one-sided limits are infinite, $x = -2$ *is a vertical asymptote.*

$$\lim_{x\to 5^-} \frac{x^2-4x+3}{x^2-3x-10} = \lim_{x\to 5^-} \frac{x^2-4x+3}{(x+2)(x-5)} = \frac{8}{(7)(-\varepsilon)} = \frac{\left(\frac{8}{7}\right)}{(-\varepsilon)} = -\infty$$

$$\begin{array}{ll} & x \to 5^- \\ \Rightarrow & x < 5 \\ \Rightarrow & x-5 < 0 \end{array}$$

$$\lim_{x\to 5^+} \frac{x^2-4x+3}{x^2-3x-10} = \lim_{x\to 5^+} \frac{x^2-4x+3}{(x+2)(x-5)} = \frac{8}{(7)(\varepsilon)} = \frac{\left(\frac{8}{7}\right)}{(\varepsilon)} = +\infty$$

$$\begin{array}{ll} & x \to 5^+ \\ \Rightarrow & x > 5 \\ \Rightarrow & x-5 > 0 \end{array}$$

Since the one-sided limits are **infinite,** $x = 5$ *is a vertical asymptote.*

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to-\infty} \frac{x^2-4x+3}{x^2-3x-10} = \lim_{x\to-\infty} \frac{x^2}{x^2} = \lim_{x\to-\infty} 1 = 1$

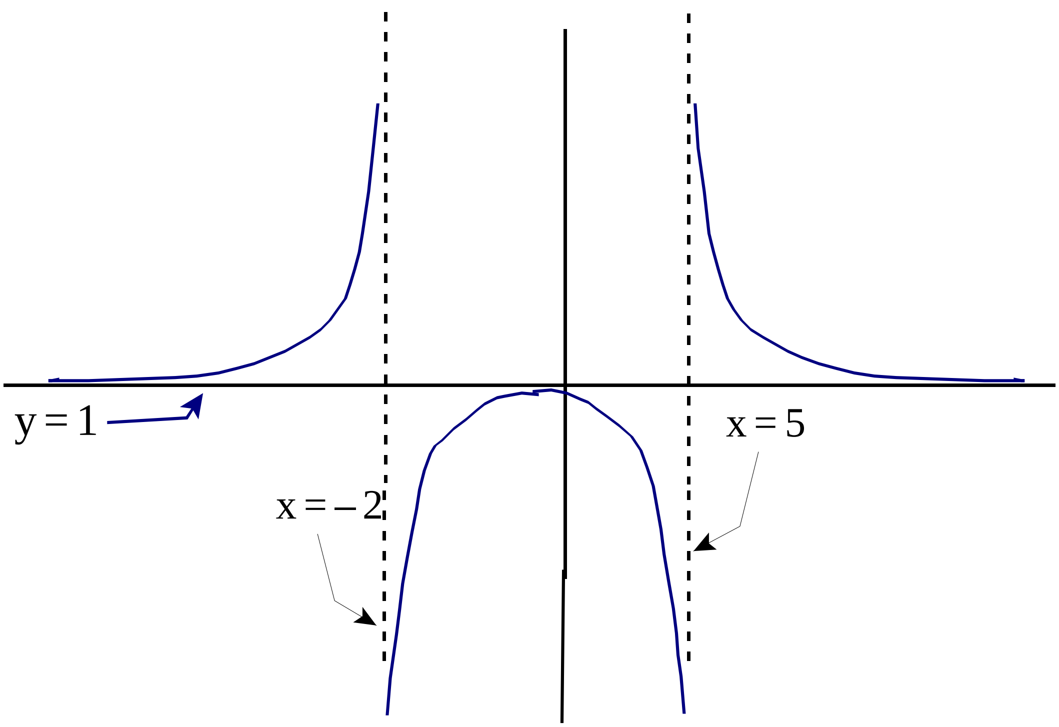
$\lim_{x\to+\infty} \frac{x^2-4x+3}{x^2-3x-10} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$

Since the limits are **finite** and **constant,** $y = 1$ *is a horizontal asymptotes.*

Summary:

$\lim_{x\to-2^-} \frac{x^2-4x+3}{x^2-3x-10} = +\infty$	
$\lim_{x\to-2^+} \frac{x^2-4x+3}{x^2-3x-10} = -\infty$	$\lim_{x\to-\infty} \frac{x^2-4x+3}{x^2-3x-10} = 1$
$\lim_{x\to5^-} \frac{x^2-4x+3}{x^2-3x-10} = -\infty$	$\lim_{x\to+\infty} \frac{x^2-4x+3}{x^2-3x-10} = 1$
$\lim_{x\to5^+} \frac{x^2-4x+3}{x^2-3x-10} = +\infty$	

Graph $f(x) = \frac{x^2+4x+3}{x^2-3x-10}$



6. Compute: $\lim_{x\to 3} \frac{\sqrt{x+22}-5}{x-3} =$

Step #1 Try Plugging in:

$$\lim_{x\to 3} \frac{\sqrt{x+22}-5}{x-3} = \frac{\sqrt{(3)+22}-5}{(3)-3} = \frac{0}{0}$$

No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 3} \frac{\sqrt{x+22}-5}{x-3} = \lim_{x\to 3} \frac{\sqrt{x+22}-5}{x-3} \cdot \frac{\sqrt{x+22}+5}{\sqrt{x+22}+5} = \lim_{x\to 3} \frac{\left(\sqrt{x+22}\right)^2-(5)^2}{(x-3)\left[\sqrt{x+22}+5\right]}$$

$$= \lim_{x\to 3} \frac{(x+22)-25}{(x-3)\left[\sqrt{x+22}+5\right]} = \lim_{x\to 3} \frac{(x-3)}{(x-3)\left[\sqrt{x+22}+5\right]} = \lim_{x\to 3} \frac{1}{\left[\sqrt{x+22}+5\right]}$$

$$= \lim_{x\to 3} \frac{1}{\left[\sqrt{(3)+22}+5\right]} = \frac{1}{[5+5]} = \frac{1}{10}$$

i.e., $\lim_{x\to 3} \frac{\sqrt{x+22}-5}{x-3} = \frac{1}{10}$

7.

$x =$	$f(x) =$
-9.1	-2.5
-90.8	-2.1
-900.3	-2.01
$-9,000.3$	-2.001
$-90,000.9$	-2.0001

$x =$	$f(x) =$
9.1	-1.5
90.8	-1.9
900.3	-1.99
$9,000.3$	-1.999
$90,000.9$	-1.9999

Based on the information in the table above, do the following:

(a) $\lim_{x\to-\infty} f(x) = -2$

(b) $\lim_{x\to+\infty} f(x) = -2$

(c) Graph $f(x)$