

# MTH 1125 Test #1 - (9 am class) - Solutions

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**Instructions.** Show CLEARLY how you arrive at your answers.

1. Compute:  $\lim_{x \rightarrow 2} \frac{x^2+6x-12}{x^2+2x+12} =$

Step #1 Try Plugging In:

$$\lim_{x \rightarrow 2} \frac{x^2+6x-12}{x^2+2x+12} = \frac{(2)^2+6(2)-12}{(2)^2+2(2)+12} = \frac{1}{5}$$

i.e.,  $\lim_{x \rightarrow 2} \frac{x^2+6x-12}{x^2+2x+12} = \frac{1}{5}$

2. Compute:  $\lim_{x \rightarrow -1} \frac{x^2-2x-3}{x^2-5x-6} =$

Step #1 Try Plugging in:

$$\lim_{x \rightarrow -1} \frac{x^2-2x-3}{x^2-5x-6} = \frac{(-1)^2-2(-1)-3}{(-1)^2-5(-1)-6} = \frac{0}{0} \quad \begin{array}{l} \text{No Good -} \\ \text{Zero Divide!} \end{array}$$

Step #2 Try Factoring and Cancelling:

$$\lim_{x \rightarrow -1} \frac{x^2-2x-3}{x^2-5x-6} = \lim_{x \rightarrow -1} \frac{(x+1)(x-3)}{(x+1)(x-6)} = \lim_{x \rightarrow -1} \frac{(x-3)}{(x-6)} = \frac{(-1)-3}{(-1)-6} = \frac{-4}{-7} = \frac{4}{7}$$

i.e.,  $\lim_{x \rightarrow -1} \frac{x^2-2x-3}{x^2-5x-6} = \frac{4}{7}$

3. Compute:  $\lim_{x \rightarrow 2} \frac{x^2-2x-15}{x^2+4x-12} =$

Step #1 Try Plugging in:

$$\lim_{x \rightarrow 2} \frac{x^2-2x-15}{x^2+4x-12} = \frac{(2)^2-2(2)-15}{(2)^2+4(2)-12} = \frac{-15}{0} \quad \begin{array}{l} \text{No Good -} \\ \text{Zero Divide!} \end{array}$$

Step #2 Try Factoring and Cancelling:

No Good! "Factoring and Cancelling" only works when Step #1 yields  $\frac{0}{0}$ .

Step #3 Analyze the one-sided limits:

$$\lim_{x \rightarrow 2^-} \frac{x^2-2x-15}{x^2+4x-12} = \lim_{x \rightarrow 2^-} \frac{x^2-2x-15}{(x+6)(x-2)} = \frac{-15}{(8)(-\varepsilon)} = \frac{\left(-\frac{15}{8}\right)}{(-\varepsilon)} = +\infty$$

$$\begin{array}{l} x \rightarrow 2^- \\ \Rightarrow x < 2 \\ \Rightarrow x - 2 < 0 \end{array}$$

$$\lim_{x \rightarrow 2^+} \frac{x^2-2x-15}{x^2+4x-12} = \lim_{x \rightarrow 2^+} \frac{x^2-2x-15}{(x+6)(x-2)} = \frac{-15}{(8)(+\varepsilon)} = \frac{\left(-\frac{15}{8}\right)}{(+\varepsilon)} = -\infty$$

$$\begin{array}{l} x \rightarrow 2^+ \\ \Rightarrow x > 2 \\ \Rightarrow x - 2 > 0 \end{array}$$

Since the one-sided limits are not equal,  $\lim_{x \rightarrow 2} \frac{x^2-2x-15}{x^2+4x-12}$  **Does Not Exist!**

4. Compute:  $\lim_{x \rightarrow -\infty} \frac{9x^3+4x-5}{x^4+4x^3-8x} =$

$$\lim_{x \rightarrow -\infty} \frac{9x^3+4x-5}{x^4+4x^3-8x} = \lim_{x \rightarrow -\infty} \frac{9x^3}{x^4} = \lim_{x \rightarrow -\infty} \frac{9}{x} = 0$$

$$\text{i.e., } \lim_{x \rightarrow -\infty} \frac{9x^3+4x-5}{x^4+4x^3-8x} = 0$$

5.  $f(x) = \frac{x^2+4x+3}{x^2-3x-10}$  Find the asymptotes and graph

Verticals

1. Find  $x$ -values that cause division by zero.

$$\Rightarrow x^2 - 3x - 10 = 0$$

$$\Rightarrow (x + 2)(x - 5) = 0$$

$\Rightarrow x = -2$  and  $x = 5$  are possible vertical asymptotes.

2. Compute the one-sided limits.

$$\lim_{x \rightarrow -2^-} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x \rightarrow -2^-} \frac{x^2+4x+3}{(x+2)(x-5)} = \frac{-1}{(-\varepsilon)(-7)} = \frac{1}{(-\varepsilon)(7)} = \frac{\left(\frac{1}{3}\right)}{-\varepsilon} = -\infty$$

$$\begin{array}{l} x \rightarrow -2^- \\ \Rightarrow x < -2 \\ \Rightarrow x + 2 < 0 \end{array}$$

$$\lim_{x \rightarrow -2^+} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x \rightarrow -2^+} \frac{x^2+4x+3}{(x+2)(x-5)} = \frac{-1}{(\varepsilon)(-7)} = \frac{1}{(\varepsilon)(7)} = \frac{\left(\frac{1}{3}\right)}{\varepsilon} = +\infty$$

$$\begin{array}{l} x \rightarrow -2^+ \\ \Rightarrow x > -2 \\ \Rightarrow x + 2 > 0 \end{array}$$

Since the one-sided limits are infinite,  $x = -2$  is a vertical asymptote.

$$\lim_{x \rightarrow 5^-} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x \rightarrow 5^-} \frac{x^2+4x+3}{(x+2)(x-5)} = \frac{48}{(7)(-\varepsilon)} = \frac{\left(\frac{48}{7}\right)}{(-\varepsilon)} = -\infty$$

$$\begin{array}{l} x \rightarrow 5^- \\ \Rightarrow x < 5 \\ \Rightarrow x - 5 < 0 \end{array}$$

$$\lim_{x \rightarrow 5^+} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x \rightarrow 5^+} \frac{x^2+4x+3}{(x+2)(x-5)} = \frac{48}{(7)(\varepsilon)} = \frac{\left(\frac{48}{7}\right)}{(\varepsilon)} = +\infty$$

$$\begin{array}{l} x \rightarrow 5^+ \\ \Rightarrow x > 5 \\ \Rightarrow x - 5 > 0 \end{array}$$

Since the one-sided limits are **infinite**,  $x = 5$  is a vertical asymptote.

Horizontals

Compute the limits as  $x \rightarrow -\infty$  and as  $x \rightarrow +\infty$

$$\lim_{x \rightarrow -\infty} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x \rightarrow -\infty} \frac{x^2}{x^2} = \lim_{x \rightarrow -\infty} 1 = 1$$

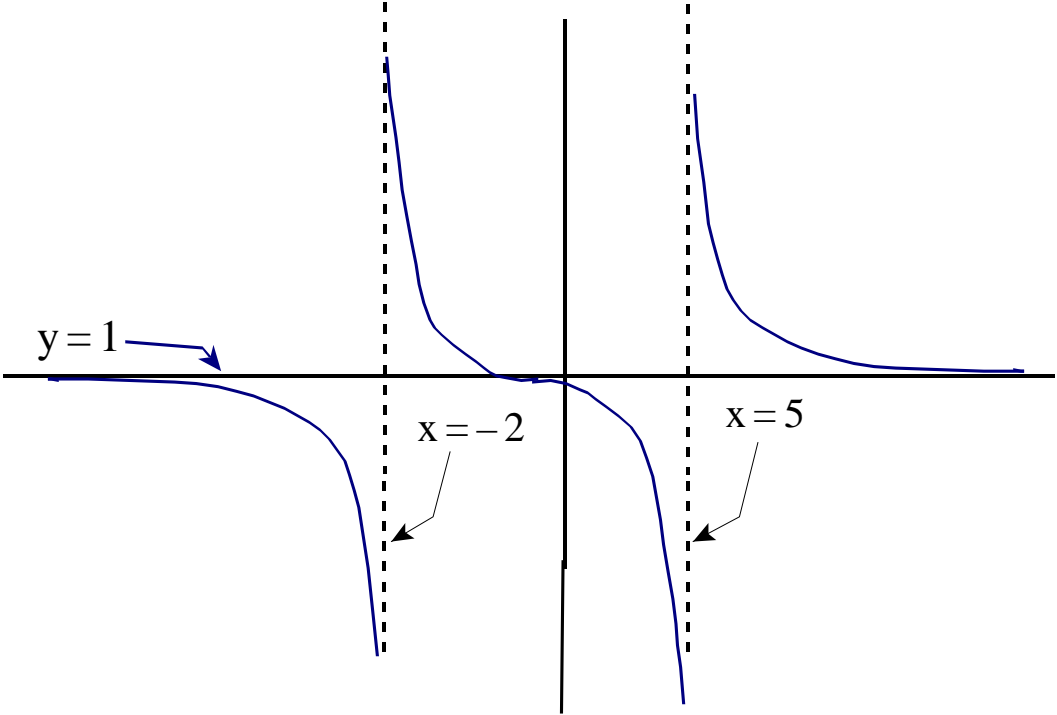
$$\lim_{x \rightarrow +\infty} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x \rightarrow +\infty} \frac{x^2}{x^2} = \lim_{x \rightarrow +\infty} 1 = 1$$

Since the limits are **finite** and **constant**,  $y = 1$  is a horizontal asymptotes.

Summary:

$\lim_{x \rightarrow -2^-} \frac{x^2+4x+3}{x^2-3x-10} = -\infty$	$\lim_{x \rightarrow -\infty} \frac{x^2+4x+3}{x^2-3x-10} = 1$
$\lim_{x \rightarrow -2^+} \frac{x^2+4x+3}{x^2-3x-10} = +\infty$	$\lim_{x \rightarrow +\infty} \frac{x^2+4x+3}{x^2-3x-10} = 1$
$\lim_{x \rightarrow 5^-} \frac{x^2+4x+3}{x^2-3x-10} = -\infty$	
$\lim_{x \rightarrow 5^+} \frac{x^2+4x+3}{x^2-3x-10} = +\infty$	

Graph  $f(x) = \frac{x^2+4x+3}{x^2-3x-10}$



6. Compute:  $\lim_{x \rightarrow 1} \frac{\sqrt{x+24}-5}{x-1} =$

Step #1 Try Plugging in:

$$\lim_{x \rightarrow 1} \frac{\sqrt{x+24}-5}{x-1} = \frac{\sqrt{(1)+24}-5}{(1)-1} = \frac{0}{0} \quad \begin{array}{l} \text{No Good -} \\ \text{Zero Divide!} \end{array}$$

Step #2 Try Factoring and Cancelling:

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{\sqrt{x+24}-5}{x-1} &= \lim_{x \rightarrow 1} \frac{\sqrt{x+24}-5}{x-1} \cdot \frac{\sqrt{x+24}+5}{\sqrt{x+24}+5} = \lim_{x \rightarrow 1} \frac{(\sqrt{x+24})^2 - (5)^2}{(x-1)[\sqrt{x+24}+5]} \\ &= \lim_{x \rightarrow 1} \frac{(x+24)-25}{(x-1)[\sqrt{x+24}+5]} = \lim_{x \rightarrow 1} \frac{(x-1)}{(x-1)[\sqrt{x+24}+5]} = \lim_{x \rightarrow 1} \frac{1}{[\sqrt{x+24}+5]} \\ &= \lim_{x \rightarrow 1} \frac{1}{[\sqrt{(1)+24}+5]} = \frac{1}{[5+5]} = \frac{1}{10} \end{aligned}$$

$\text{i.e., } \lim_{x \rightarrow 1} \frac{\sqrt{x+24}-5}{x-1} = \frac{1}{10}$
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7.

$x =$	$f(x) =$
-9.1	1.5
-90.8	1.9
-900.3	1.99
-9,000.3	1.999
-90,000.9	1.9999

$x =$	$f(x) =$
9.1	2.5
90.8	2.1
900.3	2.01
9,000.3	2.001
90,000.9	2.0001

Based on the information in the table above, do the following:

(a)  $\lim_{x \rightarrow -\infty} f(x) = 2$

(b)  $\lim_{x \rightarrow +\infty} f(x) = 2$

(c) Graph  $f(x)$

