MTH 1125 Test #1 - (1 pm class) - Solutions

Fall 2018

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 3} \frac{x^2+6x-12}{x^2+2x+12} =$

Step #1 Try Plugging In:

$$\lim_{x\to 3} \frac{x^2+6x-12}{x^2+2x+12} = \frac{(3)^2+6(3)-12}{(3)^2+2(3)+12} = \frac{15}{27} = \frac{5}{9}$$

i.e., $\lim_{x\to 3} \frac{x^2+6x-12}{x^2+2x+12} = \frac{5}{9}$

2. Compute: $\lim_{x\to 6} \frac{x^2-9x+18}{x^2-5x-6} =$

Step #1 Try Plugging in:

$$\lim_{x\to 6} \frac{x^2-9x+18}{x^2-5x-6} = \frac{(6)^2-9(6)+18}{(6)^2-5(6)-6} = \frac{0}{0}$$

No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 6} \frac{x^2-9x+18}{x^2-5x-6} = \lim_{x\to 6} \frac{(x-3)(x-6)}{(x+1)(x-6)} = \lim_{x\to 6} \frac{(x-3)}{(x+1)} = \frac{(6)-3}{(6)+1} = \frac{3}{7}$$

i.e., $\lim_{x\to 6} \frac{x^2-9x+18}{x^2-5x-6} = \frac{3}{7}$

3. Compute: $\lim_{x\to\ 2} \frac{x^2+4x-9}{x^2+4x-12} =$

Step #1 Try Plugging in:

$$\lim_{x\to\ 2} \frac{x^2+4x-9}{x^2+4x-12} = \frac{(2)^2+4(2)-9}{(2)^2+4(2)-12} = \frac{3}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

No Good! "Factoring and Cancelling" only works when Step #1 yields $\frac{0}{0}$.

Step #3 Analyze the one-sided limits:

$$\lim_{x\to\ 2^-} \frac{x^2+4x-9}{x^2+4x-12} = \lim_{x\to\ 2^-} \frac{x^2+4x-9}{(x+6)(x-2)} = \frac{3}{(8)(-\varepsilon)} = \frac{\left(\frac{3}{8}\right)}{(-\varepsilon)} = -\infty$$

$$\begin{array}{ll} & x \to 2^- \\ \Rightarrow & x < 2 \\ \Rightarrow & x - 2 < 0 \end{array}$$

$$\lim_{x\to\ 2^-} \frac{x^2+4x-9}{x^2+4x-12} = \lim_{x\to\ 2^-} \frac{x^2+4x-9}{(x+6)(x-2)} = \frac{3}{(8)(+\varepsilon)} = \frac{\left(\frac{3}{8}\right)}{(+\varepsilon)} = +\infty$$

$$\begin{array}{ll} & x \to 2^+ \\ \Rightarrow & x > 2 \\ \Rightarrow & x - 2 > 0 \end{array}$$

Since the one-sided limits are not equal, $\lim_{x\to\ 2} \frac{x^2+4x-9}{x^2+4x-12}$ **Does Not Exist!**

4. Compute: $\lim_{x\to\ -\infty} \frac{3x^6+4x^4-5}{x^4+4x^3-8x} =$

$$\lim_{x\to\ -\infty} \frac{3x^6+4x^4-5}{x^4+4x^3-8x} = \lim_{x\to\ -\infty} \frac{3x^6}{x^4} = \lim_{x\to\ -\infty} 3x^2 = +\infty$$

i.e., $\lim_{x\to\ -\infty} \frac{3x^6+4x^4-5}{x^4+4x^3-8x} = +\infty$

5. $f(x) = \frac{x^2+4x+3}{x^2-3x-10}$ Find the asymptotes and graph

Verticals

1. Find x-values that cause division by zero.

$\Rightarrow x^2 - 3x - 10 = 0$

$\Rightarrow (x+2)(x-5) = 0$

$\Rightarrow x = -2$ and $x = 5$ are possible vertical asymptotes.

2. Compute the one-sided limits.

$$\lim_{x\to -2^-} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x\to -2^-} \frac{x^2+4x+3}{(x+2)(x-5)} = \frac{-1}{(-\varepsilon)(-7)} = \frac{1}{(-\varepsilon)(7)} = \frac{\left(\frac{1}{3}\right)}{-\varepsilon} = -\infty$$

$$\begin{array}{|ll|} & x \to -2^- \\ \Rightarrow & x < -2 \\ \Rightarrow & x + 2 < 0 \\ \end{array}$$

$$\lim_{x\to -2^+} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x\to -2^+} \frac{x^2+4x+3}{(x+2)(x-5)} = \frac{-1}{(\varepsilon)(-7)} = \frac{1}{(\varepsilon)(7)} = \frac{\left(\frac{1}{3}\right)}{\varepsilon} = +\infty$$

$$\begin{array}{|ll|} & x \to -2^+ \\ \Rightarrow & x > -2 \\ \Rightarrow & x + 2 > 0 \\ \end{array}$$

Since the one-sided limits are infinite, $x = -2$ *is a vertical asymptote.*

$$\lim_{x\to 5^-} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x\to 5^-} \frac{x^2+4x+3}{(x+2)(x-5)} = \frac{48}{(7)(-\varepsilon)} = \frac{\left(\frac{48}{7}\right)}{(-\varepsilon)} = -\infty$$

$$\begin{array}{|ll|} & x \to 5^- \\ \Rightarrow & x < 5 \\ \Rightarrow & x - 5 < 0 \\ \end{array}$$

$$\lim_{x\to 5^+} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x\to 5^+} \frac{x^2+4x+3}{(x+2)(x-5)} = \frac{48}{(7)(\varepsilon)} = \frac{\left(\frac{48}{7}\right)}{(\varepsilon)} = +\infty$$

$$\begin{array}{|ll|} & x \to 5^+ \\ \Rightarrow & x > 5 \\ \Rightarrow & x - 5 > 0 \\ \end{array}$$

Since the one-sided limits are **infinite,** $x = 5$ *is a vertical asymptote.*

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to-\infty} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x\to-\infty} \frac{x^2}{x^2} = \lim_{x\to-\infty} 1 = 1$

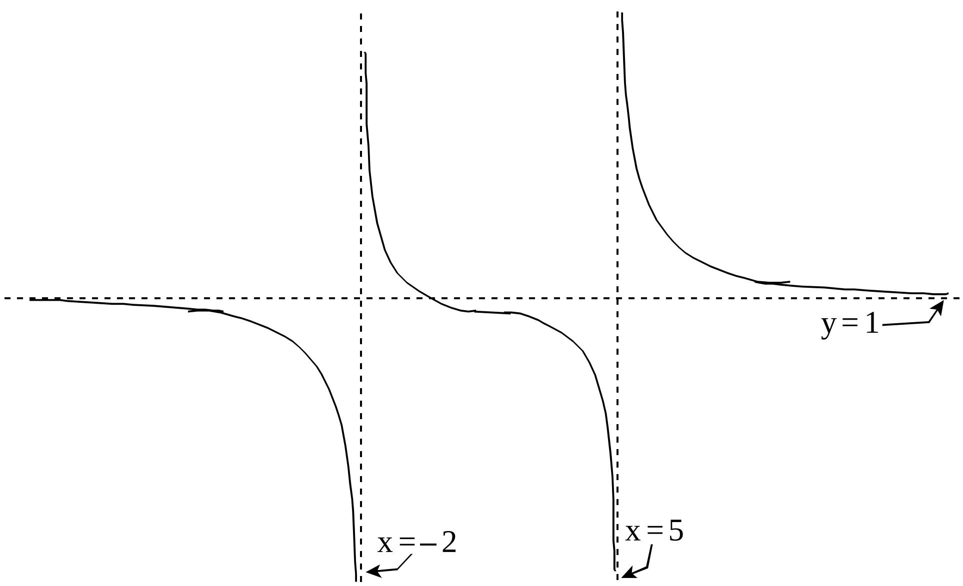
$\lim_{x\to+\infty} \frac{x^2+4x+3}{x^2-3x-10} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$

Since the limits are **finite** and **constant,** $y = 1$ *is a horizontal asymptote.*

Summary:

$\lim_{x\to-2^-} \frac{x^2+4x+3}{x^2-3x-10} = -\infty$	
$\lim_{x\to-2^+} \frac{x^2+4x+3}{x^2-3x-10} = +\infty$	$\lim_{x\to-\infty} \frac{x^2+4x+3}{x^2-3x-10} = 1$
$\lim_{x\to5^-} \frac{x^2+4x+3}{x^2-3x-10} = -\infty$	$\lim_{x\to+\infty} \frac{x^2+4x+3}{x^2-3x-10} = 1$
$\lim_{x\to5^+} \frac{x^2+4x+3}{x^2-3x-10} = +\infty$	

Graph $f(x) = \frac{x^2+4x+3}{x^2-3x-10}$



6. Compute: $\lim_{x\to 1} \frac{\sqrt{x+8}-3}{x-1} =$

Step #1 Try Plugging in:

$$\lim_{x\to 1} \frac{\sqrt{x+8}-3}{x-1} = \frac{\sqrt{(1)+8}-3}{(1)-1} = \frac{0}{0}$$

No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 1} \frac{\sqrt{x+8}-3}{x-1} = \lim_{x\to 1} \frac{\sqrt{x+8}-3}{x-1} \cdot \frac{\sqrt{x+8}+3}{\sqrt{x+8}+3} = \lim_{x\to 1} \frac{\left(\sqrt{x+8}\right)^2-(3)^2}{(x-1)\left[\sqrt{x+8}+3\right]}$$

$$= \lim_{x\to 1} \frac{(x+8)-9}{(x-1)\left[\sqrt{x+8}+3\right]} = \lim_{x\to 1} \frac{(x-1)}{(x-1)\left[\sqrt{x+8}+3\right]} = \lim_{x\to 1} \frac{1}{\left[\sqrt{x+8}+3\right]}$$

$$= \lim_{x\to 1} \frac{1}{\left[\sqrt{(1)+8}+3\right]} = \frac{1}{[3+3]} = \frac{1}{6}$$

i.e., $\lim_{x\to 1} \frac{\sqrt{x+8}-3}{x-1} = \frac{1}{6}$

7.

$x =$	$f(x) =$
-10	10.87
-100	1.12
-1000	1.009
$-10,000$	1.0023
$-100,000$	1.00009

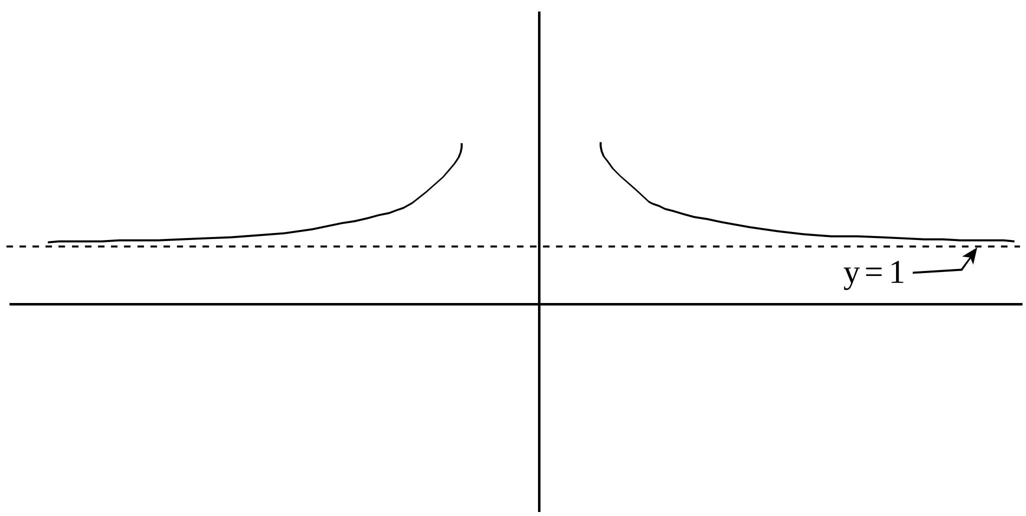
$x =$	$f(x) =$
10	10.87
100	1.12
1000	1.009
$10,000$	1.0023
$100,000$	1.00009

Based on the information in the table above, do the following:

(a) $\lim_{x \to -\infty} f(x) = 1$

(b) $\lim_{x \to +\infty} f(x) = 1$

(c) Graph $f(x)$



Extra: (Wow! 10 points) Show CLEARLY how you arrive at your answer!

Compute: $\lim_{x\to -\infty} \frac{3x+5}{\sqrt{x^2+6x+9}}$

$\lim_{x\to\,-\infty} \frac{3x+5}{\sqrt{x^2+6x+9}}$

$= \quad \lim_{x\to\,-\infty} \frac{3x}{\sqrt{x^2}}$

$\nwarrow$

$\left(\sqrt{x^2} = |x|\right)$

$\swarrow$

$= \quad \lim_{x\to\,-\infty} \frac{3x}{|x|}$

$\nwarrow$

(Since $x \to -\infty$, we can assume that $x < 0$. Hence, $|x| = -x$)

$\swarrow$

$= \quad \lim_{x\to\,-\infty} \frac{3x}{-x}$

$= \quad \lim_{x\to\,-\infty} (-3) = -3$

$\boxed{\lim_{x\to -\infty} \frac{3x+5}{\sqrt{x^2+6x+9}} = -3}$