MTH 1125 Test #1 - (2 pm class) - Solutions

Fall 2018

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 3}\frac{x^2+2x+12}{x^2+6x-12} =$

Step #1 Try Plugging In:

$\lim_{x\to 3}\frac{x^2+2x+12}{x^2+6x-12} = \frac{(3)^2+2(3)+12}{(3)^2+6(3)-12} = \frac{27}{15} = \frac{9}{5}$

i.e., $\lim_{x\to 3}\frac{x^2+2x+12}{x^2+6x-12} = \frac{9}{5}$

2. Compute: $\lim_{x\to 6}\frac{x^2-5x-6}{x^2-9x+18} =$

(a) $\lim_{x\to 6}\frac{x^2-5x-6}{x^2-9x+18} = \frac{(6)^2-5(6)-6}{(6)^2-9(6)+18} = \frac{0}{0}$ No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

$\lim_{x\to 6}\frac{x^2-5x-6}{x^2-9x+18} = \lim_{x\to 6}\frac{(x+1)(x-6)}{(x-3)(x-6)} = \lim_{x\to 6}\frac{(x+1)}{(x-3)} = \frac{(6)+1}{(6)-3} = \frac{7}{3}$

i.e., $\lim_{x\to 6}\frac{x^2-5x-6}{x^2-9x+18} = \frac{7}{3}$

3. Compute: $\lim_{x\to\ 2} \frac{x^2+4x-9}{x^2+2x-8} =$

Step #1 Try Plugging in:

$$\lim_{x\to\ 2} \frac{x^2+4x-9}{x^2+2x-8} = \frac{(2)^2+4(2)-9}{(2)^2+2(2)-8} = \frac{3}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

No Good! "Factoring and Cancelling" only works when Step #1 yields $\frac{0}{0}$.

Step #3 Analyze the one-sided limits:

$$\lim_{x\to\ 2^-} \frac{x^2+4x-9}{x^2+2x-8} = \lim_{x\to\ 2^-} \frac{x^2+4x-9}{(x+4)(x-2)} = \frac{3}{(6)(-\varepsilon)} = \frac{\left(\frac{3}{6}\right)}{(-\varepsilon)} = \frac{\left(\frac{1}{2}\right)}{(-\varepsilon)} = -\infty$$

$$\begin{array}{ll} & x \to 2^- \\ \Rightarrow & x < 2 \\ \Rightarrow & x - 2 < 0 \end{array}$$

$$\lim_{x\to\ 2^-} \frac{x^2+4x-9}{x^2+2x-8} = \lim_{x\to\ 2^-} \frac{x^2+4x-9}{(x+4)(x-2)} = \frac{3}{(6)(+\varepsilon)} = \frac{\left(\frac{3}{6}\right)}{(+\varepsilon)} = \frac{\left(\frac{1}{2}\right)}{(+\varepsilon)} = +\infty$$

$$\begin{array}{ll} & x \to 2^+ \\ \Rightarrow & x > 2 \\ \Rightarrow & x - 2 > 0 \end{array}$$

Since the one-sided limits are not equal, $\lim_{x\to\ 2} \frac{x^2+4x-9}{x^2+2x-8}$ **Does Not Exist!**

4. Compute: $\lim_{x\to\ -\infty} \frac{3x^7+4x^3-5}{x^4+4x^3-8x} =$

$$\lim_{x\to\ -\infty} \frac{3x^7+4x^3-5}{x^4+4x^3-8x} = \lim_{x\to\ -\infty} \frac{3x^7}{x^4} = \lim_{x\to\ -\infty} 3x^3 = -\infty$$

i.e., $\lim_{x\to\ -\infty} \frac{3x^7+4x^3-5}{x^4+4x^3-8x} = -\infty$

5. $f(x) = \frac{x^2-9}{x^2+3x-10}$ Find the asymptotes and graph

Verticals

1. Find x-values that cause division by zero.

$\Rightarrow x^2 + 3x - 10 = 0$

$\Rightarrow (x+5)(x-2) = 0$

$\Rightarrow x = -5$ and $x = 2$ are possible vertical asymptotes.

2. Compute the one-sided limits.

$\lim_{x\to\ -5^-} \frac{x^2-9}{x^2+3x-10} = \lim_{x\to\ -5^-} \frac{x^2-9}{(x+5)(x-2)} = \frac{16}{(-\varepsilon)(-7)} = \frac{16}{(\varepsilon)(7)} = \frac{\left(\frac{16}{7}\right)}{\varepsilon} = +\infty$

$$\boxed{\begin{array}{ll} & x \to -5^- \\ \Rightarrow & x < -5 \\ \Rightarrow & x+5 < 0 \end{array}}$$

$\lim_{x\to\ -5^+} \frac{x^2-9}{x^2+3x-10} = \lim_{x\to\ -5^+} \frac{x^2-9}{(x+5)(x-2)} = \frac{16}{(+\varepsilon)(-7)} = \frac{\left(-\frac{16}{7}\right)}{\varepsilon} = -\infty$

$$\boxed{\begin{array}{ll} & x \to -5^+ \\ \Rightarrow & x > -5 \\ \Rightarrow & x+5 > 0 \end{array}}$$

Since the one-sided limits are infinite, $x = -5$ *is a vertical asymptote.*

$\lim_{x\to\ 2^-} \frac{x^2-9}{x^2+3x-10} = \lim_{x\to\ 2^-} \frac{x^2-9}{(x+5)(x-2)} = \frac{-5}{(7)(-\varepsilon)} = \frac{\left(-\frac{5}{7}\right)}{(-\varepsilon)} = +\infty$

$$\boxed{\begin{array}{ll} & x \to 2^- \\ \Rightarrow & x < 2 \\ \Rightarrow & x-2 < 0 \end{array}}$$

$\lim_{x\to 2^+} \frac{x^2-9}{x^2-3x-10} = \lim_{x\to 2^+} \frac{x^2-9}{(x+5)(x-2)} = \frac{-5}{(7)(+\varepsilon)} = \frac{\left(-\frac{5}{7}\right)}{(\varepsilon)} = -\infty$

$$\boxed{\begin{array}{ll} & x \to 2^+ \\ \Rightarrow & x > 2 \\ \Rightarrow & x-2 > 0 \end{array}}$$

Since the one-sided limits are **infinite,** $x = 2$ *is a vertical asymptote.*

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to-\infty} \frac{x^2-9}{x^2+3x-10} = \lim_{x\to-\infty} \frac{x^2}{x^2} = \lim_{x\to-\infty} 1 = 1$

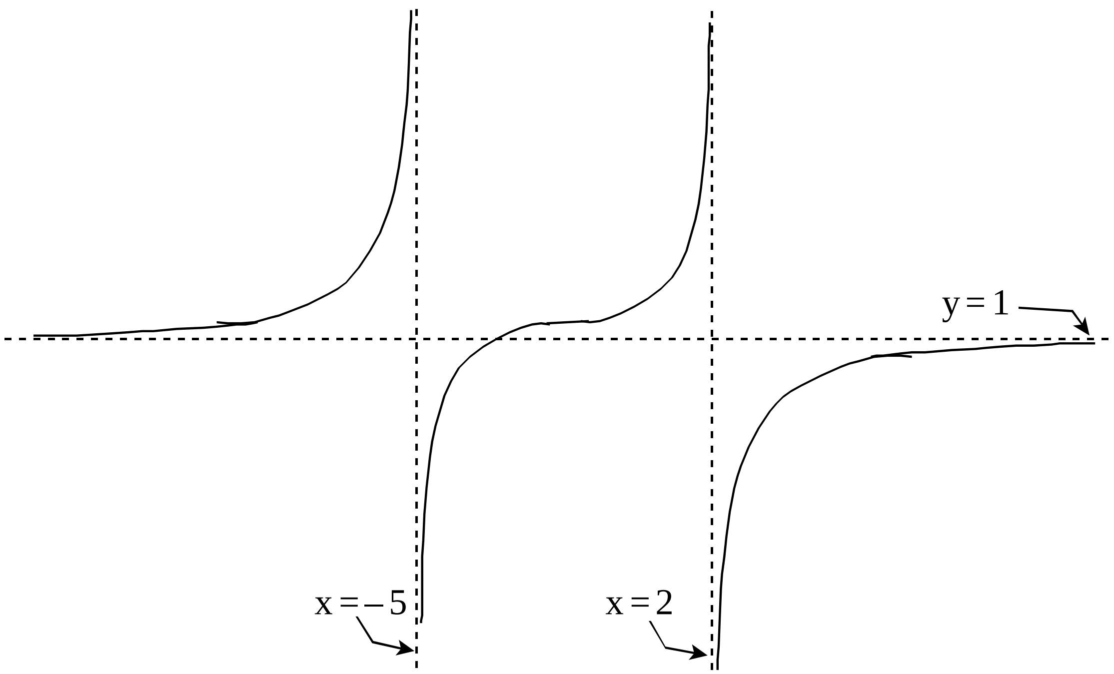
$\lim_{x\to+\infty} \frac{x^2-9}{x^2+3x-10} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$

Since the limits are **finite** and **constant,** $y = 1$ *is a horizontal asymptote.*

Summary:

$\lim_{x\to-5^-} \frac{x^2-9}{x^2+3x-10} = +\infty$	
$\lim_{x\to-5^+} \frac{x^2-9}{x^2+3x-10} = -\infty$	$\lim_{x\to-\infty} \frac{x^2-9}{x^2+3x-10} = 1$
$\lim_{x\to2^-} \frac{x^2-9}{x^2+3x-10} = +\infty$	$\lim_{x\to+\infty} \frac{x^2-9}{x^2+3x-10} = 1$
$\lim_{x\to2^+} \frac{x^2-9}{x^2+3x-10} = -\infty$	

Graph $f(x) = \frac{x^2-9}{x^2+3x-10}$



6. Compute: $\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} =$

Step #1 Try Plugging in:

$$\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} = \frac{\sqrt{(1)+3}-2}{(1)-1} = \frac{0}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} = \lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} \cdot \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2} = \lim_{x\to 1} \frac{\left(\sqrt{x+3}\right)^2-(2)^2}{(x-1)\left[\sqrt{x+3}+2\right]}$$

$$= \lim_{x\to 1} \frac{(x+3)-4}{(x-1)\left[\sqrt{x+3}+2\right]} = \lim_{x\to 1} \frac{(x-1)}{(x-1)\left[\sqrt{x+3}+2\right]} = \lim_{x\to 1} \frac{1}{\left[\sqrt{x+3}+2\right]}$$

$$= \lim_{x\to 1} \frac{1}{\left[\sqrt{(1)+3}+2\right]} = \frac{1}{[2+2]} = \frac{1}{4}$$

i.e., $\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} = \frac{1}{4}$

7.

$x =$	$f(x) =$
-2.5	3.6
-2.1	3.8
-2.01	3.9
-2.001	3.99
-2.0001	3.999

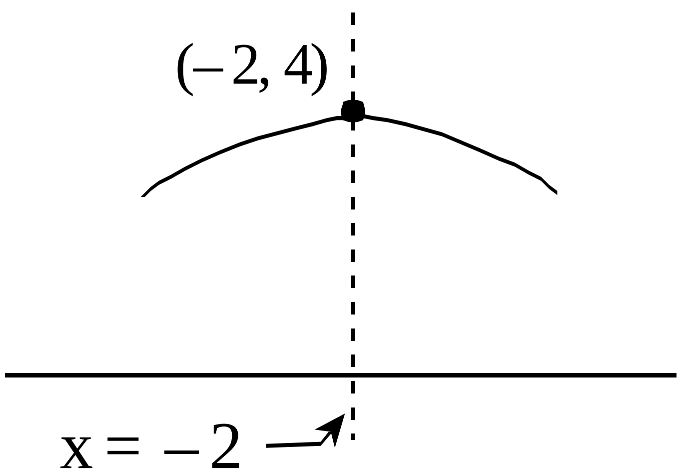
$x =$	$f(x) =$
-1.5	3.6
-1.9	3.8
-1.99	3.9
-1.999	3.99
-1.9999	3.999

Based on the information in the table above, do the following:

(a) $\lim_{x \to -2^-} f(x) = 4$

(b) $\lim_{x \to -2^+} f(x) = 4$

(c) Graph $f(x)$



Extra: (Wow - 10 points!) Show CLEARLY how you arrive at your answer!

Compute: $\lim_{x\to\,-\infty} \frac{2x+5}{\sqrt{x^2+4x+4}} =$

$\lim_{x\to\,-\infty} \frac{2x+5}{\sqrt{x^2+4x+4}}$

$= \lim_{x\to\,-\infty} \frac{2x}{\sqrt{x^2}}$

$\left(\sqrt{x^2} = |x|\right)$

$= \lim_{x\to\,-\infty} \frac{2x}{|x|}$

(Since $x \to -\infty$, we can assume that $x < 0$. Hence, $|x| = -x$)

$= \lim_{x\to\,-\infty} \frac{2x}{-x}$

$= \lim_{x\to\,-\infty} (-2) = -2$

$$\boxed{\lim_{x\to-\infty} \frac{2x+5}{\sqrt{x^2+4x+4}} = -2}$$