

# MTH 1125 - Test 2 (12pm Class) - Solutions

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**Instructions.** Show CLEARLY how you arrive at your answers.

1. Compute:  $\frac{d}{dx} \left[ \frac{1}{5}x^5 + x^4 + 3x^3 + 8x^2 + 25x + 8\sqrt{x} + 6 \right] =$

$$\begin{aligned} & \frac{d}{dx} \left[ \frac{1}{5}x^5 + x^4 + 3x^3 + 8x^2 + 25x + 8x^{\frac{1}{2}} + 6 \right] \\ &= \frac{1}{5} [5x^4] + [4x^3] + 3 [3x^2] + 16 [x] + 25 + 4x^{-\frac{1}{2}} + 0 \\ &= x^4 + 4x^3 + 9x^2 + 16x + 25 + 4x^{-\frac{1}{2}} \end{aligned}$$

i.e.,  $\frac{d}{dx} \left[ \frac{1}{5}x^5 + x^4 + 3x^3 + 8x^2 + 25x + 8\sqrt{x} + 6 \right] = x^4 + 4x^3 + 9x^2 + 16x + 25 + 4x^{-\frac{1}{2}}$

2. Compute:  $\frac{d}{dx} [x^6 \cot(x)] =$

$$\frac{d}{dx} \left[ \underbrace{x^6}_{1^{st}} \underbrace{\cot(x)}_{2^{nd}} \right] = \underbrace{6x^5}_{1^{st} \text{ prime}} \cdot \underbrace{\cot(x)}_{2^{nd}} + \underbrace{(-\csc^2(x))}_{2^{nd} \text{ prime}} \cdot \underbrace{x^6}_{1^{st}}$$

$$\frac{d}{dx} [x^6 \cot(x)] = 6x^5 \cot(x) - \csc^2(x) \cdot x^6$$

3. Compute:  $\frac{d}{dx} \left[ \frac{6x^2-12x+5}{\sin(x)} \right] =$

$$\frac{d}{dx} \left[ \frac{\overbrace{6x^2-12x+5}^{\text{top}}}{\underbrace{\sin(x)}_{\text{Bottom}}} \right] = \frac{\overbrace{(12x-12)}^{\text{top prime}} \cdot \underbrace{\sin(x)}_{\text{bottom}} - \underbrace{(\cos(x))}_{\text{bottom prime}} \cdot \overbrace{(6x^2-12x+5)}^{\text{top}}}{\underbrace{(\sin(x))^2}_{\text{bottom squared}}}$$

i.e.,  $\frac{d}{dx} \left[ \frac{6x^2-12x+5}{\sin(x)} \right] = \frac{(12x-12) \sin(x) - \cos(x)(6x^2-12x+5)}{\sin^2(x)}$

4. Compute:  $\frac{d}{dx} \left[ (4x^{15} + 2x^{10})^5 \right] =$  This is the derivative of a function raised to a power.

$$\frac{d}{dx} \left[ (4x^{15} + 2x^{10})^5 \right] = \underbrace{5 (4x^{15} + 2x^{10})^4}_{\substack{\text{power rule} \\ \text{as usual}}} \cdot \underbrace{(60x^{15} + 20x^9)}_{\substack{\text{derivative} \\ \text{of inner}}}$$

$$\text{i.e., } \frac{d}{dx} \left[ (4x^{15} + 2x^{10})^5 \right] = 5 (4x^{15} + 2x^{10})^4 (60x^{15} + 20x^9)$$

5. Given that  $f(x) = 3x^2 - 2x + 1$ , give the *equation* of the line tangent to the graph of  $f(x)$  at the point  $(1, 2)$ .

We need two things:

- i. A point on the line (We have that:  $(x_1, y_1) = (1, 2)$ )
- ii. The slope of the line (This is  $f'(x_1)$ )

$$f'(x) = 6x - 2$$

At the point  $(x_1, y_1) = (1, 2)$ , **the slope is**  $f'(2) = 6(1) - 2 = 4$

We will use the Point-Slope equation of a line:

$$y - y_1 = m(x - x_1) \quad (\text{Where } m \text{ is the slope and } (x_1, y_1) \text{ is a known point on the line.})$$

Thus, the equation of the line tangent to the graph of  $f(x)$  is:

$$y - 2 = 4(x - 1)$$

The equation of the line tangent is  $y - 2 = 4(x - 1)$

6. Given that  $y = 4x^3 + 4x^2$  and that  $x = \sec(t)$ ; compute  $\frac{dy}{dt}$  **using the Leibniz form of the Chain Rule.** (In particular, when doing this exercise, *write explicitly the Leibniz form of the chain rule that you are going to use.*)

**We know:**

$$\frac{dy}{dx} = 12x^2 + 8x$$

$$\frac{dx}{dt} = \sec(t) \tan(t)$$

**We want:**  $\frac{dy}{dt}$

By the Leibniz form of the Chain Rule:

$$\frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt} = (12x^2 + 8x) (\sec(t) \tan(t)) = \underbrace{(12(\sec(t))^2 + 8\sec(t)) \sec(t) \tan(t)}_{\text{express solely in terms of independent variable } t}$$

i.e.  $\frac{dy}{dt} = (12(\sec(t))^2 + 8\sec(t)) \sec(t) \tan(t)$

7. Compute:  $\frac{d}{dx} [\cos(5x^2 + 4x + 3)] =$

Outer:  $= \cos(\quad)$   
 Deriv. of outer  $= -\sin(\quad)$

$$\frac{d}{dx} \left[ \begin{array}{c} \cos(5x^2 + 4x + 3) \\ \uparrow \quad \uparrow \\ \text{outer} \quad \text{inner} \end{array} \right] = \underbrace{-\sin(5x^2 + 4x + 3)}_{\text{Deriv of outer, eval. at inner}} \cdot \underbrace{(10x + 4)}_{\text{deriv. of inner}}$$

i.e.,  $\frac{d}{dx} [\cos(5x^2 + 4x + 3)] = -\sin(5x^2 + 4x + 3) (10x + 4)$

8. Compute:  $\frac{d}{dx} \left[ \left( \frac{6x^2+9x}{2x^5+10x} \right)^4 \right] =$  In the broadest sense, this is the derivative of a *function raised to a power* - USE the CHAIN RULE.

$$\begin{aligned} \frac{d}{dx} \left[ \underbrace{\left( \frac{6x^2+9x}{2x^5+10x} \right)^4}_{(g(x))^n} \right] &= 4 \underbrace{\left( \frac{6x^2+9x}{2x^5+10x} \right)^3}_{\text{power rule as usual}} \cdot \underbrace{\left( \frac{d}{dx} \left[ \frac{6x^2+9x}{2x^5+10x} \right] \right)}_{\text{deriv of inner Function}} \\ &= 4 \left( \frac{6x^2+9x}{2x^5+10x} \right)^3 \underbrace{\frac{(12x+9)(2x^5+10x) - (10x^4+10)(6x^2+9x)}{(2x^5+10x)^2}}_{\text{quotient rule}} \end{aligned}$$

i.e.,  $\frac{d}{dx} \left[ \left( \frac{6x^2+9x}{2x^5+10x} \right)^4 \right] = 4 \left( \frac{6x^2+9x}{2x^5+10x} \right)^3 \cdot \frac{(12x+9)(2x^5+10x) - (10x^4+10)(6x^2+9x)}{(2x^5+10x)^2}$

9. Compute:  $\frac{d}{dx} [\tan^{10}(2x^5+10x)] =$  Re-write!

$\frac{d}{dx} [(\tan(2x^5+10x))^{10}]$  This is the derivative of a function, raised to a power

$$\begin{aligned} \frac{d}{dx} [(\tan(2x^5+10x))^{10}] &= \underbrace{10(\tan(2x^5+10x))^9}_{\text{power rule as usual}} \cdot \underbrace{\left( \frac{d}{dx} [\tan(2x^5+10x)] \right)}_{\text{derivative of inner}} \\ &= 10(\tan(2x^5+10x))^9 \cdot \underbrace{(\sec^2(2x^5+10x)) \cdot (10x+10)}_{\text{Chain Rule}} \end{aligned}$$

i.e.,  $\frac{d}{dx} [\tan^{10}(2x^5+10x)] = 10(\tan(2x^5+10x))^9 (\sec^2(2x^5+10x)) \cdot (10x+10)$

10. Given that  $S'(x) = \frac{1}{2S(x)}$ ; compute  $\frac{d}{dx} [S(\sin(x))]$

Outer: = $S(\quad)$
Deriv. of outer = $\frac{1}{2S(\quad)}$

$$\frac{d}{dx} \left[ S \left( \underbrace{\sin(x)}_{\substack{\uparrow \\ \text{inner}}} \right) \right] = \underbrace{\frac{1}{2S(\sin(x))}}_{\substack{\text{Deriv of outer,} \\ \text{eval. at inner}}} \cdot \underbrace{\cos(x)}_{\substack{\text{deriv. of} \\ \text{inner}}} = \frac{\cos(x)}{2S(\sin(x))}$$

outer    inner

i.e., $\frac{d}{dx} [S(\sin(x))] = \frac{1}{2S(\sin(x))} \cdot \cos(x) = \frac{\cos(x)}{2S(\sin(x))}$
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11. Given that  $f(x) = 8x^2 - 2x + 3$ , compute  $f'(x)$  **using the definition of derivative.** (i.e., using the “limit process.”)

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{[8(x+\Delta x)^2 - 2(x+\Delta x) + 3] - [8x^2 - 2x + 3]}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{[8(x^2 + 2x\Delta x + \Delta x^2) - 2(x+\Delta x) + 3] - [8x^2 - 2x + 3]}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{[8x^2 + 16x\Delta x + 8\Delta x^2 - 2x - 2\Delta x + 3] - [8x^2 - 2x + 3]}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{16x\Delta x + 8\Delta x^2 - 2\Delta x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(16x + 8\Delta x - 2)}{\Delta x} = \lim_{\Delta x \rightarrow 0} (16x + 8\Delta x - 2) = 16x + 8(0) - 2 = 16x - 2 \end{aligned}$$

i.e., $f'(x) = 16x - 2$
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