

MTH 1125 - Test 2 (9am Class) - Solutions

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Name _____

Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\frac{d}{dx} [5x^5 + 4x^4 + 3x^3 + 2x^2 + x + 4\sqrt{x} + 2] =$

$$\begin{aligned} & \frac{d}{dx} [5x^5 + 4x^4 + 3x^3 + 2x^2 + x + 4x^{\frac{1}{2}} + 2] \\ &= 5[5x^4] + 4[4x^3] + 3[3x^2] + 2[2x] + 2x^{-\frac{1}{2}} + 0 \\ &= 25x^4 + 16x^3 + 9x^2 + 4x + 2x^{-\frac{1}{2}} \end{aligned}$$

i.e., $\frac{d}{dx} [5x^5 + 4x^4 + 3x^3 + 2x^2 + x + 4x^{\frac{1}{2}} + 2] = 25x^4 + 16x^3 + 9x^2 + 4x + 2x^{-\frac{1}{2}}$

2. Compute: $\frac{d}{dx} [x^5 \tan(x)] =$

$$\frac{d}{dx} \left[\underbrace{x^5}_{1^{st}} \underbrace{\tan(x)}_{2^{nd}} \right] = \underbrace{5x^4}_{1^{st} \text{ prime}} \cdot \underbrace{\tan(x)}_{2^{nd}} + \underbrace{\sec^2(x)}_{2^{nd} \text{ prime}} \cdot \underbrace{x^5}_{1^{st}}$$

$$\frac{d}{dx} [x^5 \tan(x)] = 5x^4 \tan(x) + \sec^2(x) \cdot x^5$$

3. Compute: $\frac{d}{dx} \left[\frac{3x^2 - 6x + 2}{\cos(x)} \right] =$

$$\frac{d}{dx} \left[\frac{\overbrace{3x^2 - 6x + 2}^{\text{top}}}{\underbrace{\cos(x)}_{\text{Bottom}}} \right] = \frac{\overbrace{(6x - 6)}^{\text{top prime}} \cdot \underbrace{\cos(x)}_{\text{bottom}} - \underbrace{(-\sin(x))}_{\text{bottom prime}} \cdot \overbrace{(3x^2 - 6x + 2)}^{\text{top}}}{\underbrace{(\cos(x))^2}_{\text{bottom squared}}}$$

i.e., $\frac{d}{dx} \left[\frac{3x^2 - 6x + 2}{\cos(x)} \right] = \frac{(6x - 6) \cos(x) + \sin(x)(3x^2 - 6x + 2)}{\cos^2(x)}$

4. Compute: $\frac{d}{dx} \left[(6x^{20} + 8x^{10})^5 \right] =$ This is the derivative of a function raised to a power.

$$\frac{d}{dx} \left[(6x^{20} + 8x^{10})^5 \right] = \underbrace{5 (6x^{20} + 8x^{10})^4}_{\substack{\text{power rule} \\ \text{as usual}}} \cdot \underbrace{(120x^{19} + 80x^4)}_{\substack{\text{derivative} \\ \text{of inner}}}$$

i.e., $\frac{d}{dx} \left[(6x^{20} + 8x^{10})^5 \right] = 5 (6x^{20} + 8x^{10})^4 (120x^{19} + 80x^4)$

5. Given that $f(x) = 2x^2 - 2x + 1$, give the *equation* of the line tangent to the graph of $f(x)$ at the point $(2, 5)$.

We need two things:

- i. A point on the line (We have that: $(x_1, y_1) = (2, 5)$)
- ii. The slope of the line (This is $f'(x_1)$)

$$f'(x) = 4x - 2$$

At the point $(x_1, y_1) = (2, 5)$, **the slope is** $f'(2) = 4(2) - 2 = 6$

We will use the Point-Slope equation of a line:

$$y - y_1 = m(x - x_1) \quad (\text{Where } m \text{ is the slope and } (x_1, y_1) \text{ is a known point on the line.})$$

Thus, the equation of the line tangent to the graph of $f(x)$ is:

$$y - 5 = 6(x - 2)$$

The equation of the line tangent is $y - 5 = 6(x - 2)$

6. Given that $y = 3x^2 + 6x$ and that $x = \csc(t)$; compute $\frac{dy}{dt}$ **using the Leibniz form of the Chain Rule.** (In particular, when doing this exercise, *write explicitly the Leibniz form of the chain rule that you are going to use.*)

We know:

$$\frac{dy}{dx} = 6x + 6$$

$$\frac{dx}{dt} = -\csc(t) \cot(t)$$

We want: $\frac{dy}{dt}$

By the Leibniz form of the Chain Rule:

$$\frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt} = (6x + 6) (-\csc^2(t) \cot(t)) = \underbrace{(6 \csc(t) + 6) (-\csc^2(t) \cot(t))}_{\text{express solely in terms of independent variable } t}$$

i.e. $\frac{dy}{dt} = (6 \csc(t) + 6) (-\csc^2(t) \cot(t))$

7. Compute: $\frac{d}{dx} [\sin(4x^2 + 8x + 3)] =$

Outer: $= \sin(\quad)$
 Deriv. of outer $= \cos(\quad)$

$$\frac{d}{dx} \left[\begin{array}{c} \sin \left(\underbrace{4x^2 + 8x + 3}_{\substack{\uparrow \quad \uparrow \\ \text{outer} \quad \text{inner}}} \right) \\ \hline \cos \left(\underbrace{4x^2 + 8x + 3}_{\substack{\text{Deriv of outer,} \\ \text{eval. at inner}}} \right) \cdot \underbrace{(8x + 8)}_{\substack{\text{deriv. of} \\ \text{inner}}} \end{array} \right] =$$

i.e., $\frac{d}{dx} [\sin(4x^2 + 8x + 3)] = \cos(4x^2 + 8x + 3) (8x + 8)$

8. Compute: $\frac{d}{dx} \left[\left(\frac{2x^5+10x}{3x^3+9x} \right)^5 \right] =$ In the broadest sense, this is the derivative of a *function raised to a power* - USE the CHAIN RULE.

$$\begin{aligned} \frac{d}{dx} \left[\underbrace{\left(\frac{2x^5+10x}{3x^3+9x} \right)^5}_{(g(x))^n} \right] &= \underbrace{5 \left(\frac{2x^5+10x}{3x^3+9x} \right)^4}_{\text{power rule as usual}} \cdot \underbrace{\left(\frac{d}{dx} \left[\frac{2x^5+10x}{3x^3+9x} \right] \right)}_{\text{deriv of inner Function}} \\ &= 5 \left(\frac{2x^5+10x}{3x^3+9x} \right)^4 \underbrace{\frac{(10x^4+10)(3x^3+9x) - (9x^2+9)(2x^5+10x)}{(3x^3+9x)^2}}_{\text{quotient rule}} \end{aligned}$$

i.e., $\frac{d}{dx} \left[\left(\frac{2x^5+10x}{3x^3+9x} \right)^5 \right] = 5 \left(\frac{2x^5+10x}{3x^3+9x} \right)^4 \cdot \frac{(10x^4+10)(3x^3+9x) - (9x^2+9)(2x^5+10x)}{(3x^3+9x)^2}$

9. Compute: $\frac{d}{dx} [\cot^5(4x^2+8x)] =$ Re-write!

$\frac{d}{dx} [(\cot(4x^2+8x))^5]$ This is the derivative of a function, raised to a power

$$\begin{aligned} \frac{d}{dx} [(\cot(4x^2+8x))^5] &= \underbrace{5(\cot(4x^2+8x))^4}_{\text{power rule as usual}} \cdot \underbrace{\left(\frac{d}{dx} [\cot(4x^2+8x)] \right)}_{\text{derivative of inner}} \\ &= 5(\cot(4x^2+8x))^4 \cdot \underbrace{(-\csc^2(4x^2+8x)) \cdot (8x+8)}_{\text{Chain Rule}} \end{aligned}$$

i.e., $\frac{d}{dx} [\cot^5(4x^2+8x)] = 5(\cot(4x^2+8x))^4 (-\csc^2(4x^2+8x)) \cdot (8x+8)$

10. Given that $S'(x) = \frac{1}{2S(x)}$; compute $\frac{d}{dx} [S(x^2)]$

Outer: = $S(\quad)$
Deriv. of outer = $\frac{1}{2S(\quad)}$

$$\frac{d}{dx} \left[S \left(\underbrace{x^2}_{\substack{\uparrow \\ \text{inner}}} \right) \right] = \underbrace{\frac{1}{2S(x^2)}}_{\substack{\text{Deriv of outer,} \\ \text{eval. at inner}}} \cdot \underbrace{\frac{2x}{\substack{\uparrow \\ \text{deriv. of} \\ \text{inner}}}} = \frac{2x}{2S(x^2)} = \frac{x}{S(x^2)}$$

i.e., $\frac{d}{dx} [S(x^2)] = \frac{2x}{2S(x^2)} = \frac{x}{S(x^2)}$

11. Given that $f(x) = 4x^2 - 3x + 2$, compute $f'(x)$ **using the definition of derivative.** (i.e., using the “limit process.”)

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{[4(x+\Delta x)^2 - 3(x+\Delta x) + 2] - [4x^2 - 3x + 2]}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{[4(x^2 + 2x\Delta x + \Delta x^2) - 3(x+\Delta x) + 2] - [4x^2 - 3x + 2]}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{[4x^2 + 8x\Delta x + 4\Delta x^2 - 3x - 3\Delta x + 2] - [4x^2 - 3x + 2]}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{8x\Delta x + 4\Delta x^2 - 3\Delta x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(8x + 4\Delta x - 3)}{\Delta x} = \lim_{\Delta x \rightarrow 0} (8x + 4\Delta x - 3) = 8x + 4(0) - 3 = 8x - 3 \end{aligned}$$

i.e., $f'(x) = 8x - 3$
