# MTH 1125 - Test 2 (1pm Class) - Solutions

FALL 2018

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**Instructions.** Show CLEARLY how you arrive at your answers.

1. Compute: $\frac{d}{dx}\left[3x^5+5x^4+8x^3+10x^2+15x+10\sqrt{x}+2\right]=$

$\frac{d}{dx}\left[3x^5+5x^4+8x^3+10x^2+15x+10x^{\frac{1}{2}}+2\right]$

$=3\left[5x^4\right]+5\left[4x^3\right]+8\left[3x^2\right]+10\left[2x\right]+15+10\left[\frac{1}{2}x^{-\frac{1}{2}}\right]+0$

$=15x^4+20x^3+24x^2+20x+15+5x^{-\frac{1}{2}}$

i.e., $\frac{d}{dx}\left[3x^5+5x^4+8x^3+10x^2+15x+10\sqrt{x}+2\right]=15x^4+20x^3+24x^2+20x+15+5x^{-\frac{1}{2}}$

2. Compute: $\frac{d}{dx}\left[\left(x^3+3x\right)\tan(x)\right]=$

$$\frac{d}{dx}\left[\underbrace{\left(x^3+3x\right)}_{1^{st}}\underbrace{\tan(x)}_{2^{nd}}\right]=\underbrace{\left(3x^2+3\right)}_{1^{\text{st}}\text{ prime}}\cdot\underbrace{\tan(x)}_{2^{nd}}+\underbrace{\sec^2(x)}_{2^{\text{nd}}\text{ prime}}\cdot\underbrace{\left(x^3+3x\right)}_{1^{st}}$$

$\frac{d}{dx}\left[\left(x^3+3x\right)\tan(x)\right]=\left(3x^2+3\right)\tan(x)+\sec^2(x)\left(x^3+3x\right)$

3. Compute: $\frac{d}{dx}\left[\frac{3x^2-6x+2}{4x^2+3x+3}\right]=$

$$\frac{d}{dx}\left[\frac{\overbrace{3x^2-6x+2}^{\text{top}}}{\underbrace{4x^2+3x+3}_{\text{Bottom}}}\right]=\frac{\overbrace{(6x-6)}^{\text{top prime}}\cdot\overbrace{\left(4x^2+3x+3\right)}^{\text{bottom}}-\overbrace{(8x+3)}^{\text{bottom prime}}\cdot\overbrace{\left(3x^2-6x+2\right)}^{\text{top}}}{\underbrace{\left(4x^2+3x+3\right)^2}_{\substack{\text{bottom}\\\text{squared}}}}$$

i.e., $\frac{d}{dx}\left[\frac{3x^2-6x+2}{4x^2+3x+3}\right]=\frac{(6x-6)\left(4x^2+3x+3\right)-(8x+3)\left(3x^2-6x+2\right)}{\left(4x^2+3x+3\right)^2}$

4. Compute: $\frac{d}{dx}\left[\left(6x^{10}+\tan(x)\right)^5\right] =$ This is the derivative of a function raised to a power.

$$\frac{d}{dx}\left[\left(6x^{10}+\tan(x)\right)^5\right] = \underbrace{5\left(6x^{10}+\tan(x)\right)^4}_{\substack{\text{power rule}\\ \text{as usual}}} \cdot \underbrace{\left(60x^9+\sec^2(x)\right)}_{\substack{\text{derivative}\\ \text{of inner}}}$$

i.e., $\frac{d}{dx}\left[\left(6x^{10}+\tan(x)\right)^5\right] = 5\left(6x^{10}+\tan(x)\right)^4\left(60x^9+\sec^2(x)\right)$

5. Given that $f(x) = 4x^2+2x+2$, give the *equation* of the line tangent to the graph of $f(x)$ at the point $(2, 22)$.

We need two things:

i. A point on the line (We have that: $(x_1, y_1) = (2, 22)$)

ii. The slope of the line (This is $f'(x_1)$)

$f'(x) = 8x+2$

At the point $(x_1, y_1) = (2, 22)$, **the slope is** $f'(2) = 8(2)+2 = 18$

We will use the Point-Slope equation of a line:

$y-y_1 = m(x-x_1)$ (Where $m$ is the slope and $(x_1, y_1)$ is a known point on the line.)

Thus, the equation of the line tangent to the graph of $f(x)$ is:

$y-22 = 18(x-2)$

The equation of the line tangent is $y-22 = 18(x-2)$

6. Given that $y = \frac{1}{2}x^2 + 4x$ and that $x = \csc(t)$; compute $\frac{dy}{dt}$ **using the Liebniz form of the Chain Rule.** (In particular, when doing this exercise, *write explicitly the Liebniz form of the chain rule that you are going to use.)*

**We know:**

$\frac{dy}{dx} = x + 4$

$\frac{dx}{dt} = -\csc(t)\cot(t)$

**We want:** $\frac{dy}{dt}$

By the Liebniz form of the Chain Rule:

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = (x+4)(-\csc(t)\cot(t)) = \underbrace{(\csc(t)+4)(-\csc(t)\cot(t))}_{\text{express solely in terms of independent variable } t}$$

i.e. $\frac{dy}{dt} = (\csc(t)+4)(-\csc(t)\cot(t))$

7. Compute: $\frac{d}{dx}\left[\sin\left(5x^3 + 8x^2 + 3\right)\right] =$

| | | |
|---|---|---|
| Outer: | = | sin (   ) |
| Deriv. of outer | = | cos (   ) |

$$\frac{d}{dx}\left[\underset{\substack{\uparrow \\ \text{outer}}}{\sin}\Big(\underset{\substack{\uparrow \\ \text{inner}}}{\underbrace{5x^3 + 8x^2 + 3}}\Big)\right] = \underbrace{\cos\left(5x^3 + 8x^2 + 3\right)}_{\text{Deriv of outer, eval. at inner}} \cdot \underbrace{\left(15x^2 + 16\right)}_{\text{deriv. of inner}}$$

i.e., $\frac{d}{dx}\left[\sin\left(5x^3 + 8x^2 + 3\right)\right] = \cos\left(5x^3 + 8x^2 + 3\right)\left(15x^2 + 16\right)$

8. Compute: $\frac{d}{dx}\left[\left(\frac{2x^4+8x}{3x^4+12x}\right)^4\right] =$ In the broadest sense, this is the derivative of a *function raised to a power* - USE the CHAIN RULE.

$$\frac{d}{dx}\left[\underbrace{\left(\frac{2x^4+8x}{3x^4+12x}\right)^4}_{(g(x))^n}\right] = \underbrace{4\left(\frac{2x^4+8x}{3x^4+12x}\right)^3}_{\text{power rule as usual}} \cdot \underbrace{\left(\frac{d}{dx}\left[\frac{2x^4+8x}{3x^4+12x}\right]\right)}_{\text{deriv of inner Function}}$$

$$= 4\left(\frac{2x^4+8x}{3x^4+12x}\right)^3 \underbrace{\frac{(8x^3+8)(3x^4+12x) - (12x^3+12)(2x^4+8x)}{(3x^4+12x)^2}}_{\text{quotient rule}}$$

i.e., $\frac{d}{dx}\left[\left(\frac{2x^4+8x}{3x^4+12x}\right)^4\right] = 4\left(\frac{2x^4+8x}{3x^4+12x}\right)^3 \cdot \frac{(8x^3+8)(3x^4+12x) - (12x^3+12)(2x^4+8x)}{(3x^4+12x)^2}$

9. Compute: $\frac{d}{dx}\left[\tan^5(3x^3+9x)\right] =$ Re-write!

$\frac{d}{dx}\left[(\tan(3x^3+9x))^5\right]$ This is the derivative of a function, raised to a power

$$\frac{d}{dx}\left[(\tan(3x^3+9x))^5\right] = \underbrace{5\left(\tan\left(3x^3+9x\right)\right)^4}_{\text{power rule as usual}} \cdot \underbrace{\left(\frac{d}{dx}\left[\tan\left(3x^3+9x\right)\right]\right)}_{\text{derivative of inner}}$$

$$= 5\left(\tan(3x^3+9x)\right)^4 \cdot \underbrace{\left(\sec^2\left(3x^3+9x\right)\right)\cdot(9x+9)}_{\text{Chain Rule}}$$

i.e., $\frac{d}{dx}\left[\tan^5(3x^3+9x)\right] = 5\left(\tan(3x^3+9x)\right)^4 \sec^2(3x^3+9x)(9x+9)$

10. Given that $L'(x) = \frac{1}{x}$ (i.e., $\frac{d}{dx}[L(x)] = \frac{1}{x}$); compute $\frac{d}{dx}[L(x^2)]$

| | | |
|---|---|---|
| Outer: | $=$ | $L(\quad)$ |
| Deriv. of outer | $=$ | $\frac{1}{(\quad)}$ |

$$\frac{d}{dx}\Big[S\big(\underbrace{x^2}\big)\Big] = \underbrace{\frac{1}{x^2}}_{\text{Deriv of outer, eval. at inner}} \cdot \underbrace{2x}_{\text{deriv. of inner}} = \frac{2x}{x^2} = \frac{2}{x}$$

↑ outer ↑ inner

i.e., $\frac{d}{dx}[L(x^2)] = \frac{2x}{x^2} = \frac{2}{x}$

11. Given that $f(x) = 3x^2 - 4x + 5$, compute $f'(x)$ **using the definition of derivative.** (i.e., using the "limit process.")

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left[3(x+\Delta x)^2 - 4(x+\Delta x) + 5\right] - \left[3x^2 - 4x + 5\right]}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\left[3\left(x^2 + 2x\Delta x + \Delta x^2\right) - 4(x+\Delta x) + 5\right] - \left[3x^2 - 4x + 5\right]}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\left[3x^2 + 6x\Delta x + 3\Delta x^2 - 4x - 4\Delta x + 5\right] - \left[3x^2 - 4x + 5\right]}{\Delta x} = \lim_{\Delta x \to 0} \frac{6x\Delta x + 3\Delta x^2 - 4\Delta x}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\Delta x(6x + 3\Delta x - 4)}{\Delta x} = \lim_{\Delta x \to 0} (6x + 3\Delta x - 4) = 6x + 3(0) - 4 = 6x - 4$

i.e., $f'(x) = 6x - 4$

12. Given that $x^3 + y^3 = \sin(y)$, compute $y'$

i. Differentiate both sides w.r.t. $x$

$\frac{d}{dx}\left[x^3 + y^3\right] = \frac{d}{dx}\left[\sin(y)\right]$

$\Rightarrow 3x^2 + 3y^2 \cdot y' = \cos(y) \cdot y'$

ii. Solve algebraically for $y'$

a. Get $y'$ terms on left side, all other terms on right side

$\Rightarrow 3y^2 \cdot y' - \cos(y) \cdot y' = -3x^2$

b. Factor out $y'$

$\Rightarrow \left(3y^2 - \cos(y)\right) y' = -3x^2$

c. Divide both sides by the cofactor of $y'$

$y' = \frac{-3x^2}{3y^2 - \cos(y)} = -\frac{3x^2}{3y^2 - \cos(y)}$

$$\boxed{y' = -\frac{3x^2}{3y^2 - \cos(y)}}$$