

# MTH 1125 - Test 2 (2pm Class) - Solutions

FALL 2018

Pat Rossi

Name \_\_\_\_\_

**Instructions.** Show CLEARLY how you arrive at your answers.

1. Compute:  $\frac{d}{dx} [4x^5 + 6x^4 + 9x^3 + 12x^2 + 20x + 16\sqrt{x} + 2] =$

$$\frac{d}{dx} [4x^5 + 6x^4 + 9x^3 + 12x^2 + 20x + 16\sqrt{x} + 2]$$

$$= 4 [5x^4] + 6 [4x^3] + 9 [3x^2] + 12 [2x] + 20 + 16 \left[ \frac{1}{2}x^{-\frac{1}{2}} \right] + 0$$

$$= 20x^4 + 24x^3 + 27x^2 + 24x + 20 + 8x^{-\frac{1}{2}}$$

i.e.,  $\frac{d}{dx} [4x^5 + 6x^4 + 9x^3 + 12x^2 + 20x + 16\sqrt{x} + 2] = 20x^4 + 24x^3 + 27x^2 + 24x + 20 + 8x^{-\frac{1}{2}}$

2. Compute:  $\frac{d}{dx} [(x^4 + 4x) \sec(x)] =$

$$\frac{d}{dx} \left[ \underbrace{(x^4 + 4x)}_{1^{st}} \underbrace{\sec(x)}_{2^{nd}} \right] = \underbrace{(4x^3 + 4)}_{1^{st} \text{ prime}} \cdot \underbrace{\sec(x)}_{2^{nd}} + \underbrace{\sec(x) \tan(x)}_{2^{nd} \text{ prime}} \cdot \underbrace{(x^4 + 4x)}_{1^{st}}$$

$$\frac{d}{dx} [(x^4 + 4x) \sec(x)] = (4x^3 + 4) \sec(x) + \sec(x) \tan(x) (x^4 + 4x)$$

3. Compute:  $\frac{d}{dx} \left[ \frac{4x^2+3x+3}{3x^2-6x+2} \right] =$

$$\frac{d}{dx} \left[ \frac{\overbrace{4x^2 + 3x + 3}^{\text{top}}}{\underbrace{3x^2 - 6x + 2}_{\text{Bottom}}} \right] = \frac{\overbrace{(8x + 3)}^{\text{top prime}} \cdot \overbrace{(3x^2 - 6x + 2)}^{\text{bottom}} - \overbrace{(6x - 6)}^{\text{bottom prime}} \cdot \overbrace{(4x^2 + 3x + 3)}^{\text{top}}}{\underbrace{(3x^2 - 6x + 2)^2}_{\text{bottom squared}}}$$

i.e.,  $\frac{d}{dx} \left[ \frac{4x^2+3x+3}{3x^2-6x+2} \right] = \frac{(8x+3)(3x^2-6x+2) - (6x-6)(4x^2+3x+3)}{(3x^2-6x+2)^2}$

4. Compute:  $\frac{d}{dx} \left[ (6x^5 + \sec(x))^{10} \right] =$  This is the derivative of a function raised to a power.

$$\frac{d}{dx} \left[ (6x^5 + \sec(x))^{10} \right] = \underbrace{10 (6x^5 + \sec(x))^9}_{\substack{\text{power rule} \\ \text{as usual}}} \cdot \underbrace{(30x^4 + \sec(x) \tan(x))}_{\substack{\text{derivative} \\ \text{of inner}}}$$

i.e.,  $\frac{d}{dx} \left[ (6x^5 + \sec(x))^{10} \right] = 10 (6x^5 + \sec(x))^9 (30x^4 + \sec(x) \tan(x))$

5. Given that  $f(x) = 2x^2 + 4x + 3$ , give the *equation* of the line tangent to the graph of  $f(x)$  at the point  $(2, 19)$ .

We need two things:

- i. A point on the line (We have that:  $(x_1, y_1) = (2, 19)$ )
- ii. The slope of the line (This is  $f'(x_1)$ )

$$f'(x) = 4x + 4$$

At the point  $(x_1, y_1) = (2, 19)$ , **the slope is**  $f'(2) = 4(2) + 4 = 12$

We will use the Point-Slope equation of a line:

$$y - y_1 = m(x - x_1) \quad (\text{Where } m \text{ is the slope and } (x_1, y_1) \text{ is a known point on the line.})$$

Thus, the equation of the line tangent to the graph of  $f(x)$  is:

$$y - 19 = 12(x - 2)$$

The equation of the line tangent is  $y - 19 = 12(x - 2)$

6. Given that  $y = \csc(t)$  and that  $t = \frac{1}{2}x^2 + 4x$ ; compute  $\frac{dy}{dx}$  **using the Leibniz form of the Chain Rule.** (In particular, when doing this exercise, *write explicitly the Leibniz form of the chain rule that you are going to use.*)

**We know:**

$$\frac{dy}{dt} = -\csc(t) \cot(t)$$

$$\frac{dt}{dx} = x + 4$$

**We want:**  $\frac{dy}{dx}$

By the Leibniz form of the Chain Rule:

$$\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = -\csc(t) \cot(t) (x + 4) = \underbrace{-\csc\left(\frac{1}{2}x^2 + 4x\right) \cot\left(\frac{1}{2}x^2 + 4x\right)}_{\text{express solely in terms of independent variable } x} (x + 4)$$

i.e.  $\frac{dy}{dx} = -\csc\left(\frac{1}{2}x^2 + 4x\right) \cot\left(\frac{1}{2}x^2 + 4x\right) (x + 4)$

7. Compute:  $\frac{d}{dx} [\cos(5x^3 + 8x^2 + 3)] =$

Outer:  $= \cos(\quad)$   
 Deriv. of outer  $= -\sin(\quad)$

$$\frac{d}{dx} \left[ \cos\left(\underbrace{5x^3 + 8x^2 + 3}_{\substack{\uparrow \quad \uparrow \\ \text{outer} \quad \text{inner}}}\right) \right] = \underbrace{-\sin(5x^3 + 8x^2 + 3)}_{\substack{\text{Deriv of outer,} \\ \text{eval. at inner}}} \cdot \underbrace{(15x^2 + 16x)}_{\substack{\text{deriv. of} \\ \text{inner}}}$$

i.e.,  $\frac{d}{dx} [\cos(5x^3 + 8x^2 + 3)] = -\sin(5x^3 + 8x^2 + 3) (15x^2 + 16x)$

8. Compute:  $\frac{d}{dx} \left[ \left( \frac{3x^4+12x}{2x^4+8x} \right)^6 \right] =$  In the broadest sense, this is the derivative of a *function raised to a power* - USE the CHAIN RULE.

$$\begin{aligned} \frac{d}{dx} \left[ \underbrace{\left( \frac{3x^4+12x}{2x^4+8x} \right)^6}_{(g(x))^n} \right] &= \underbrace{6 \left( \frac{3x^4+12x}{2x^4+8x} \right)^5}_{\text{power rule as usual}} \cdot \underbrace{\left( \frac{d}{dx} \left[ \frac{3x^4+12x}{2x^4+8x} \right] \right)}_{\text{deriv of inner Function}} \\ &= 6 \left( \frac{3x^4+12x}{2x^4+8x} \right)^5 \underbrace{\frac{(12x^3+12)(2x^4+8x) - (8x^3+8)(3x^4+12x)}{(2x^4+8x)^2}}_{\text{quotient rule}} \end{aligned}$$

i.e.,  $\frac{d}{dx} \left[ \left( \frac{3x^4+12x}{2x^4+8x} \right)^6 \right] = 6 \left( \frac{3x^4+12x}{2x^4+8x} \right)^5 \frac{(12x^3+12)(2x^4+8x) - (8x^3+8)(3x^4+12x)}{(2x^4+8x)^2}$

9. Compute:  $\frac{d}{dx} [\csc^5(2x^4+8x)] =$  In the broadest sense, this is the derivative of a *function raised to a power* - USE the CHAIN RULE. Re-write!

$\frac{d}{dx} [(\csc(2x^4+8x))^5]$  This is the derivative of a function, raised to a power

$$\begin{aligned} \frac{d}{dx} [(\csc(2x^4+8x))^5] &= \underbrace{5 (\csc(2x^4+8x))^4}_{\text{power rule as usual}} \cdot \underbrace{\left( \frac{d}{dx} \csc(2x^4+8x) \right)}_{\text{derivative of inner}} \\ &= 5 (\csc(2x^4+8x))^4 \cdot \underbrace{[-\csc(2x^4+8x) \cot(2x^4+8x) \cdot (8x+8)]}_{\text{Chain Rule}} \\ &= -5 (\csc(2x^4+8x))^4 \cdot \underbrace{\csc(2x^4+8x) \cot(2x^4+8x) \cdot (8x+8)}_{\text{Chain Rule}} \end{aligned}$$

i.e.,  $\frac{d}{dx} [(\csc(2x^4+8x))^5] = -5 (\csc(2x^4+8x))^4 \csc(2x^4+8x) \cot(2x^4+8x) (8x+8)$



12. Given that  $x^4 + y^4 = \sin(y)$ , compute  $y'$

i. Differentiate both sides w.r.t.  $x$

$$\frac{d}{dx} [x^4 + y^4] = \frac{d}{dx} [\sin(y)]$$

$$\Rightarrow 4x^3 + 4y^3 \cdot y' = \cos(y) \cdot y'$$

ii. Solve algebraically for  $y'$

a. Get  $y'$  terms on left side, all other terms on right side

$$\Rightarrow 4y^3 \cdot y' - \cos(y) \cdot y' = -4x^3$$

b. Factor out  $y'$

$$\Rightarrow (4y^3 - \cos(y)) y' = -4x^3$$

c. Divide both sides by the cofactor of  $y'$

$$y' = \frac{-4x^3}{4y^3 - \cos(y)} = -\frac{4x^3}{4y^3 - \cos(y)}$$

$$y' = -\frac{4x^3}{4y^3 - \cos(y)}$$