MTH 1125 (12pm Class) - Solutions

FALL 2017

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\int \left(12x^3 + 9x^2 - 6x + 3 + \frac{9}{2}\sqrt{x}\right) dx =$

$\int \left(12x^3 + 9x^2 - 6x + 3 + \frac{9}{2}\sqrt{x}\right) dx = \int \left(12x^3 + 9x^2 - 6x + 3 + \frac{9}{2}x^{\frac{1}{2}}\right) dx$

$= 12\left[\frac{x^4}{4}\right] + 9\left[\frac{x^3}{3}\right] - 6\left[\frac{x^2}{2}\right] + 3x + \frac{9}{2}\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C = 3x^4 + 3x^3 - 3x^2 + 3x + \frac{9}{2}\left(\frac{2}{3}\right)x^{\frac{3}{2}} + C$

$= 3x^4 + 3x^3 - 3x^2 + 3x + 3x^{\frac{3}{2}} + C$

i.e., $\int \left(12x^3 + 9x^2 - 6x + 3 + \frac{9}{2}\sqrt{x}\right) dx = 3x^4 + 3x^3 - 3x^2 + 3x + 3x^{\frac{3}{2}} + C$

2. Compute: $\int \left(8x^2+16x+4\right)^5 (4x+4)\, dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\left(8x^2+16x+4\right)^5$ (A function raised to a power is always a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(8x^2+16x+4\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(8x^2+16x+4\right)}_{\text{function}} - - - - \rightarrow \underbrace{(4x+4)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(8x^2+16x+4\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 8x^2+16x+4 \\ \Rightarrow \frac{du}{dx} &= 16x+16 \\ \Rightarrow du &= (16x+16)\,dx \\ \Rightarrow \tfrac{1}{4}du &= (4x+4)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\left(8x^2+16x+4\right)^5}_{u^5} \underbrace{(4x+4)\,dx}_{\frac{1}{4}du} = \int u^5 \tfrac{1}{4} du = \tfrac{1}{4}\int u^5 du$$

4. Integrate (in terms of u).

$$\tfrac{1}{4}\int u^5 du = \tfrac{1}{4}\left[\tfrac{u^6}{6}\right] + C = \tfrac{1}{24}u^6 + C$$

5. Re-express in terms of the original variable, x.

$$\int \left(8x^2+16x+4\right)^5 (4x+4)\,dx = \underbrace{\frac{1}{24}\left(8x^2+16x+4\right)^6 + C}_{\frac{1}{24}u^6+C}$$

i.e., $\int \left(8x^2+16x+4\right)^5 (4x+4)\,dx = \frac{1}{24}\left(8x^2+16x+4\right)^6 + C$

3. Compute: $\int \left(7\cos(x) - 3\csc^2(x) + 3\sec(x)\tan(x)\right) dx =$

$\int \left(7\cos(x) - 3\csc^2(x) + 3\sec(x)\tan(x)\right) dx = 7\left[\sin(x)\right] - 3\left[-\cot(x)\right] + 3\left[\sec(x)\right] + C$

$= 7\sin(x) + 3\cot(x) + 3\sec(x) + C$

i.e., $\int \left(7\cos(x) - 3\csc^2(x) + 3\sec(x)\tan(x)\right) dx = 7\sin(x) + 3\cot(x) + 3\sec(x) + C$

4. Compute: $\int \cos\left(3x^5+5x\right)\left(3x^4+1\right)dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\cos\left(3x^5+5x\right)$

   outer ↗ ↖ inner

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(3x^5+5x\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(3x^5+5x\right)}_{\text{function}} - - - - \rightarrow \underbrace{\left(3x^4+1\right)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(3x^5+5x\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 3x^5+5x \\ \Rightarrow \frac{du}{dx} &= 15x^4+5 \\ \Rightarrow du &= \left(15x^4+5\right)dx \\ \Rightarrow \frac{1}{5}du &= \left(3x^4+1\right)dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\cos\left(3x^5+5x\right)}_{\cos(u)}\underbrace{\left(3x^4+1\right)dx}_{\frac{1}{5}du} = \int \cos(u)\,\frac{1}{5}du = \frac{1}{5}\int \cos(u)\,du$$

4. Integrate (in terms of u).

$$\frac{1}{5}\int \cos(u)\,du = \frac{1}{5}\left[\sin(u)\right] + C = \frac{1}{5}\sin(u) + C$$

5. Re-express in terms of the original variable, x.

$$\int \cos\left(3x^5+5x\right)\left(3x^4+1\right)dx = \underbrace{\frac{1}{5}\sin\left(3x^5+5x\right)+C}_{\frac{1}{5}\sin(u)+C}$$

i.e., $\int \cos\left(3x^5+5x\right)\left(3x^4+1\right)dx = \frac{1}{5}\sin\left(3x^5+5x\right)+C$

5. $f(x) = \frac{1}{2}x^4 + 2x^3 - 9x^2 - 6x + 6$. [1]Determine the intervals on which $f(x)$ is concave up/concave down and [2]Identify the points of inflection.

i. Compute $f''(x)$ and find possible points of inflection

$f'(x) = 2x^3 + 6x^2 - 18x - 6$

$f''(x) = 6x^2 + 12x - 18$

a. "Type a" $(f''(c) = 0)$

Set $f''(x) = 6x^2 + 12x - 18 = 0$

$\Rightarrow 6x^2 + 12x - 18 = 0$

$\Rightarrow x^2 + 2x - 3 = 0$

$\Rightarrow (x+3)(x-1) = 0$

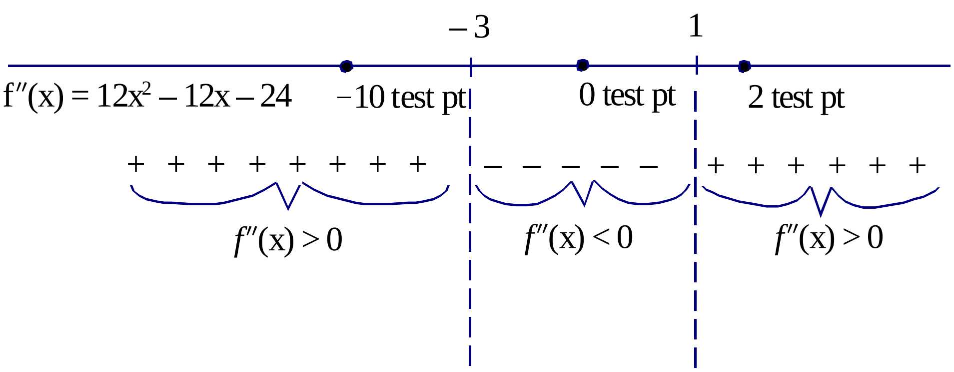
$\Rightarrow x = -3$; and $x = 1$ possible points of inflection

b. "Type b" $(f''(c)$ undefined$)$

There are none.

ii. Draw a sign graph of $f''(x)$, using the possible points of inflection to partition the x-axis

iii. From each interval select a "test point" to plug into $f''(x)$



$f(x)$ is **concave up** on the intervals $(-\infty, -3)$ and $(1, \infty)$

(Because $f''(x) > 0$ on these intervals)

$f(x)$ is **concave down** on the interval $(-3, 1)$

(Because $f''(x) < 0$ on this interval)

Since $f(x)$ changes concavity at $x = -3$ and $x = 1$, the points:

$(-3, f(-3)) = \left(-3, -\frac{141}{2}\right)$

and

$(1, f(1)) = \left(1, -\frac{13}{2}\right)$ *are* points of inflection.

6. Draw a graph of $f(x)$, given that $f(x)$ has the properties given in the table below:

$f'(x) > 0$ on the interval $(-\infty, -2)$		$\lim_{x\to-\infty} f(x) = -\infty$
	$f''(x) > 0$ on the interval $(-\infty, -2)$	
$f'(x) < 0$ on the interval $(-2, 0)$		
	$f''(x) < 0$ on the interval $(-2, 2)$	
$f'(x) > 0$ on the interval $(0, 2)$		
	$f''(x) > 0$ on the interval $(2, \infty)$	
$f'(x) < 0$ on the interval $(2, \infty)$		$\lim_{x\to+\infty} f(x) = -\infty$

First, it would be good for us to analyze what the signs of the first and second derivatives tell us about $f(x)$.

First Derivative:

$f'(x) > 0$ on the interval $(-\infty, -2)$ tells us that $f(x)$ is **increasing** on the interval $(-\infty, -2)$

$f'(x) < 0$ on the interval $(-2, 0)$ tells us that $f(x)$ is **decreasing** on the interval $(-2, 0)$

$f'(x) > 0$ on the interval $(0, 2)$ tells us that $f(x)$ is **increasing** on the interval $(0, 2)$

$f'(x) < 0$ on the interval $(2, \infty)$ tells us that $f(x)$ is **decreasing** on the interval $(2, \infty)$

Second Derivative:

$f''(x) > 0$ on the interval $(-\infty, -2)$ tells us that $f(x)$ is **concave up** on the interval $(-\infty, -2)$

$f''(x) < 0$ on the interval $(-2, 2)$ tells us that $f(x)$ is **concave down** on the interval $(-2, 2)$

$f''(x) > 0$ on the interval $(2, \infty)$ tells us that $f(x)$ is **concave up** on the interval $(2, \infty)$

Next: We will use the x-values at which $f(x)$ changes increasing/decreasing or concavity to partition the x-y plane.

Then: We will write, at the top of each interval, what it is that $f(x)$ does on that interval.

Finally: we will use this information to sketch a graph of $f(x)$.