MTH 1125 (9am Class) Test #4 - Solutions

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\int \left(16x^3 + 12x^2 - 8x + 4 + 6\sqrt{x}\right) dx =$

$\int \left(16x^3 + 12x^2 - 8x + 4 + 6\sqrt{x}\right) dx = \int \left(16x^3 + 12x^2 - 8x + 4 + 6x^{\frac{1}{2}}\right) dx$

$= 16\left[\frac{x^4}{4}\right] + 12\left[\frac{x^3}{3}\right] - 8\left[\frac{x^2}{2}\right] + 4x + 6\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C = 4x^4 + 4x^3 - 4x^2 + 4x + 6\left(\frac{2}{3}\right)x^{\frac{3}{2}} + C$

$= 4x^4 + 4x^3 - 4x^2 + 4x + 4x^{\frac{3}{2}} + C$

i.e., $\int \left(16x^3 + 12x^2 - 8x + 4 + 6\sqrt{x}\right) dx = 4x^4 + 4x^3 - 4x^2 + 4x + 4x^{\frac{3}{2}} + C$

2. Compute: $\int \left(6x^2+12x+4\right)^7 (4x+4)\,dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\left(6x^2+12x+4\right)^7$ (A function raised to a power is always a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(6x^2+12x+4\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(6x^2+12x+4\right)}_{\text{function}} ----\rightarrow \underbrace{(4x+4)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(6x^2+12x+4\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 6x^2+12x+4 \\ \Rightarrow \frac{du}{dx} &= 12x+12 \\ \Rightarrow du &= (12x+12)\,dx \\ \Rightarrow \frac{1}{3}du &= (4x+4)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\left(6x^2+12x+4\right)^7}_{u^7}\underbrace{(4x+4)\,dx}_{\frac{1}{3}du} = \int u^7 \tfrac{1}{3}du = \tfrac{1}{3}\int u^7 du$$

4. Integrate (in terms of u).

$$\tfrac{1}{3}\int u^7 du = \tfrac{1}{3}\left[\tfrac{u^8}{8}\right] + C = \tfrac{1}{24}u^8 + C$$

5. Re-express in terms of the original variable, x.

$$\int \left(6x^2+12x+4\right)^7 (4x+4)\,dx = \underbrace{\frac{1}{24}\left(6x^2+12x+4\right)^8 + C}_{\frac{1}{24}u^8+C}$$

$$\text{i.e., } \int \left(6x^2+12x+4\right)^7 (4x+4)\,dx = \tfrac{1}{24}\left(6x^2+12x+4\right)^8 + C$$

3. Compute: $\int \left(3\sin(x) - 5\sec^2(x) + 4\csc(x)\cot(x)\right) dx =$

$\int \left(3\sin(x) - 5\sec^2(x) + 4\csc(x)\cot(x)\right) dx = 3\left[-\cos(x)\right] - 5\left[\tan(x)\right] + 4\left[-\csc(x)\right] + C$

$= -3\cos(x) - 5\tan(x) - 4\csc(x) + C$

i.e., $\int \left(3\sin(x) - 5\sec^2(x) + 4\csc(x)\cot(x)\right) dx = -3\cos(x) - 5\tan(x) - 4\csc(x) + C$

4. Compute: $\int \cos\left(5x^3+9x\right)\left(5x^2+3\right)dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   $$\text{Yes!}\quad \cos\left(5x^3+9x\right)$$

   outer ↗ ↖ inner

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(5x^3+9x\right)$

   b. Is there an (approximate) function/derivative pair?

   $$\text{Yes!}\quad \underbrace{\left(5x^3+9x\right)}_{\text{function}} - - - - \longrightarrow \underbrace{\left(5x^2+3\right)}_{\text{deriv}}$$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(5x^3+9x\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{array}{rcl} u & = & 5x^3+9x \\ \Rightarrow \frac{du}{dx} & = & 15x^2+9 \\ \Rightarrow du & = & \left(15x^2+9\right)dx \\ \Rightarrow \frac{1}{3}du & = & \left(5x^2+3\right)dx \end{array}$$

3. Analyze in terms of u and du

$$\int \underbrace{\cos\left(5x^3+9x\right)}_{\cos(u)}\underbrace{\left(5x^2+3\right)dx}_{\frac{1}{3}du} = \int \cos(u)\ \frac{1}{3}du = \frac{1}{3}\int \cos(u)\,du$$

4. Integrate (in terms of u).

$$\frac{1}{3}\int \cos(u)\,du = \frac{1}{3}\left[\sin(u)\right] + C = \frac{1}{3}\sin(u) + C$$

5. Re-express in terms of the original variable, x.

$$\int \cos\left(5x^3+9x\right)\left(5x^2+3\right)dx = \underbrace{\frac{1}{3}\sin\left(5x^3+9x\right)+C}_{\frac{1}{3}\sin(u)+C}$$

i.e., $\int \cos\left(5x^3+9x\right)\left(5x^2+3\right)dx = \frac{1}{3}\sin\left(5x^3+9x\right)+C$

5. $f(x) = x^4 - 2x^3 - 12x^2 - 6x + 6$. [1]Determine the intervals on which $f(x)$ is concave up/concave down and [2]Identify the points of inflection.

   i. Compute $f''(x)$ and find possible points of inflection

   $f'(x) = 4x^3 - 6x^2 - 24x - 10$

   $f''(x) = 12x^2 - 12x - 24$

   a. "Type a" $(f''(c) = 0)$

      Set $f''(x) = 12x^2 - 12x - 24 = 0$

      $\Rightarrow 12x^2 - 12x - 24 = 0$

      $\Rightarrow x^2 - x - 2 = 0$

      $\Rightarrow (x+1)(x-2) = 0$

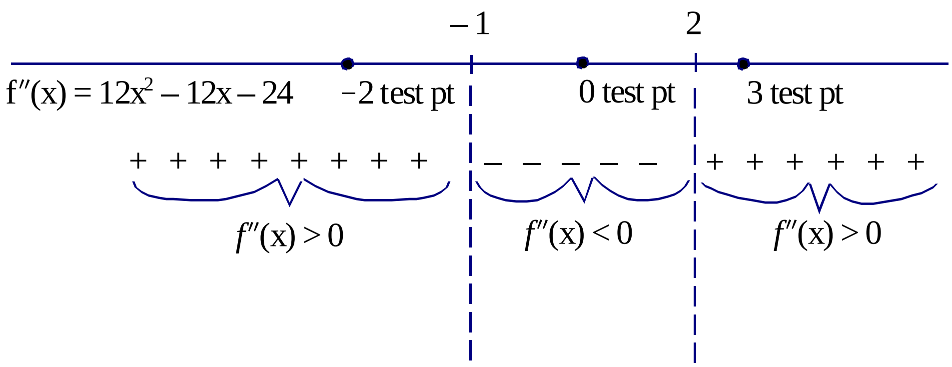
      $\Rightarrow x = -1$; and $x = 2$ possible points of inflection

   b. "Type b" $(f''(c)$ undefined$)$

      There are none.

   ii. Draw a sign graph of $f''(x)$, using the possible points of inflection to partition the x-axis

   iii. From each interval select a "test point" to plug into $f''(x)$



   $f(x)$ is **concave up** on the intervals $(-\infty, -1)$ and $(2, \infty)$

   (Because $f''(x) > 0$ on these intervals)

   $f(x)$ is **concave down** on the interval $(-1, 2)$

   (Because $f''(x) < 0$ on this interval)

   Since $f(x)$ changes concavity at $x = -1$ and $x = 2$, the points:

   $(-1, f(-1)) = (-1, 3)$

   and

   $(2, f(2)) = (2, -54)$ *are* points of inflection.

6. Sketch a graph of $f(x)$ if the following conditions hold:

$f'(x) < 0$ on the interval $(-\infty, -3)$		$\lim_{x\to -\infty} f(x) = +\infty$
	$f''(x) > 0$ on the interval $(-\infty, -2)$	
$f'(x) > 0$ on the interval $(-3, 0)$		
	$f''(x) < 0$ on the interval $(-2, 2)$	
$f'(x) < 0$ on the interval $(0, 3)$		
	$f''(x) > 0$ on the interval $(2, \infty)$	
$f'(x) > 0$ on the interval $(3, \infty)$		$\lim_{x\to +\infty} f(x) = +\infty$

First, it would be good for us to analyze what the signs of the first and second derivatives tell us about $f(x)$.

First Derivative:

$f'(x) < 0$ on the interval $(-\infty, -3)$ tells us that $f(x)$ is **decreasing** on the interval $(-\infty, -3)$

$f'(x) > 0$ on the interval $(-3, 0)$ tells us that $f(x)$ is **increasing** on the interval $(-3, 0)$

$f'(x) < 0$ on the interval $(0, 3)$ tells us that $f(x)$ is **decreasing** on the interval $(0, 3)$

$f'(x) > 0$ on the interval $(3, \infty)$ tells us that $f(x)$ is **increasing** on the interval $(3, \infty)$

Second Derivative:

$f''(x) > 0$ on the interval $(-\infty, -2)$ tells us that $f(x)$ is **concave up** on the interval $(-\infty, -2)$

$f''(x) < 0$ on the interval $(-2, 2)$ tells us that $f(x)$ is **concave down** on the interval $(-2, 2)$

$f''(x) > 0$ on the interval $(2, \infty)$ tells us that $f(x)$ is **concave up** on the interval $(2, \infty)$

Next: We will use the x-values at which $f(x)$ changes increasing/decreasing or concavity to partition the x-y plane.

Then: We will write, at the top of each interval, what it is that $f(x)$ does on that interval.

Finally: we will use this information to sketch a graph of $f(x)$.