

MTH 1125 1pm Class - Test #4 - Solutions

FALL 2018

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Name _____

Show **CLEARLY** how you arrive at your answers!

1. **Compute:** $\int (12x^3 + 9x^2 + 6x + 3 + \sqrt{x}) dx =$

$$\int (12x^3 + 9x^2 + 6x + 3 + \sqrt{x}) dx = \int (12x^3 + 9x^2 + 6x + 3 + x^{\frac{1}{2}}) dx$$

$$= 12 \left[\frac{x^4}{4} \right] + 9 \left[\frac{x^3}{3} \right] + 6 \left[\frac{x^2}{2} \right] + 3x + \frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C = 3x^4 + 3x^3 + 3x^2 + 3x + \frac{2}{3}x^{\frac{3}{2}} + C$$

i.e., $\int (12x^3 + 9x^2 + 6x + 3 + \sqrt{x}) dx = 3x^4 + 3x^3 + 3x^2 + 3x + \frac{2}{3}x^{\frac{3}{2}} + C$

2. **Compute:** $\int (6x^2 + 8x)^9 (3x + 2) dx =$

1. Is u-sub appropriate?

a. Is there a composite function?

Yes! $(6x^2 + 8x)^9$ (A function raised to a power is *always* a composite function!)

Let $u =$ the “inner” of the composite function

$$\Rightarrow u = (6x^2 + 8x)$$

b. Is there an (approximate) function/derivative pair?

Yes! $\underbrace{(6x^2 + 8x)}_{\text{function}} \text{ --- } \rightarrow \underbrace{(3x + 2)}_{\text{deriv}}$

Let $u =$ the “function” of the function/deriv pair

$$\Rightarrow u = (6x^2 + 8x)$$

c. Is the “function” of the function/deriv pair the same as the “inner” of the composite function?

(i.e., do criteria **a** and **b** suggest the same choice of u ?)

Yes! \Rightarrow u-substitution is appropriate

2. Compute du

$\begin{aligned} u &= 6x^2 + 8x \\ \Rightarrow \frac{du}{dx} &= 12x + 8 \\ \Rightarrow du &= (12x + 8) dx \\ \Rightarrow \frac{1}{4} du &= (3x + 2) dx \end{aligned}$

3. Analyze in terms of u and du

$$\int \underbrace{(6x^2 + 8x)^9}_{u^9} \underbrace{(3x + 2) dx}_{\frac{1}{4} du} = \int u^9 \frac{1}{4} du = \frac{1}{4} \int u^9 du$$

4. Integrate (in terms of u).

$$\frac{1}{4} \int u^9 du = \frac{1}{4} \left[\frac{u^{10}}{10} \right] + C = \frac{1}{40} u^{10} + C$$

5. Re-express in terms of the original variable, x .

$$\int (6x^2 + 8x)^9 (3x + 2) dx = \frac{1}{40} \underbrace{(6x^2 + 8x)^{10} + C}_{\frac{1}{40} u^{10} + C}$$

$\text{i.e., } \int (6x^2 + 8x)^9 (3x + 2) dx = \frac{1}{40} (6x^2 + 8x)^{10} + C$
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3. **Compute:** $\int (3 \sin(x) + 4 \csc(x) \cot(x)) dx =$

$$\begin{aligned} \int (3 \sin(x) + 4 \csc(x) \cot(x)) dx &= 3[-\cos(x)] + 4[-\csc(x)] + C \\ &= -3 \cos(x) - 4 \csc(x) + C \end{aligned}$$

i.e., $\int (3 \sin(x) + 4 \csc(x) \cot(x)) dx = -3 \cos(x) - 4 \csc(x) + C$
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4. **Compute:** $\int \cos(4x^2 + 12x + 4)(2x + 3) dx =$

1. Is u-sub appropriate?

a. Is there a composite function?

Yes! $\cos(4x^2 + 12x + 4)$

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Let $u =$ the “inner” of the composite function

$\Rightarrow u = (4x^2 + 12x + 4)$

b. Is there an (approximate) function/derivative pair?

Yes! $\underbrace{(4x^2 + 12x + 4)}_{\text{function}} - - - - \rightarrow \underbrace{(2x + 3)}_{\text{deriv}}$

Let $u =$ the “function” of the function/deriv pair

$\Rightarrow u = (4x^2 + 12x + 4)$

c. Is the “function” of the function/deriv pair the same as the “inner” of the composite function?

(i.e., do criteria **a** and **b** suggest the same choice of u ?)

Yes! \Rightarrow u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 4x^2 + 12x + 4 \\ \Rightarrow \frac{du}{dx} &= 8x + 12 \\ \Rightarrow du &= (8x + 12) dx \\ \Rightarrow \frac{1}{4} du &= (2x + 3) dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\cos(4x^2 + 12x + 4)}_{\cos(u)} \underbrace{(2x + 3) dx}_{\frac{1}{4} du} = \int \cos(u) \frac{1}{4} du = \frac{1}{4} \int \cos(u) du$$

4. Integrate (in terms of u).

$$\frac{1}{4} \int \cos(u) du = \frac{1}{4} [\sin(u)] + C = \frac{1}{4} \sin(u) + C$$

5. Re-express in terms of the original variable, x .

$$\int \cos(4x^2 + 12x + 4)(2x + 3) dx = \underbrace{\frac{1}{4} \sin(4x^2 + 12x + 4) + C}_{\frac{1}{4} \sin(u) + C}$$

i.e., $\int \cos(4x^2 + 12x + 4)(2x + 3) dx = \frac{1}{4} \sin(4x^2 + 12x + 4) + C$

5. **Compute:** $\int_{-1}^1 (3x^2 + 4x + 5) dx =$

$$\begin{aligned} \int_{-1}^1 \underbrace{(3x^2 + 4x + 5)}_{f(x)} dx &= \underbrace{\left[3\frac{x^3}{3} + 4\frac{x^2}{2} + 5x \right]_{-1}^1}_{F(x)} = \underbrace{\left[x^3 + 2x^2 + 5x \right]_{-1}^1}_{F(x)} = \\ &= \underbrace{\left[(1)^3 + 2(1)^2 + 5(1) \right]}_{F(1)} - \underbrace{\left[(-1)^3 + 2(-1)^2 + 5(-1) \right]}_{F(-1)} = 8 - (-4) = 12 \end{aligned}$$

i.e., $\int_{-1}^1 (3x^2 + 4x + 5) dx = 12$

6. **Compute:** $\int_0^1 (2x^2 + 1)^3 x dx =$

1. Is u-sub appropriate?

a. Is there a composite function?

Yes! $(2x^2 + 1)^3$ (A function raised to a power is *always* a composite function!)

Let $u =$ the “inner” of the composite function

$$\Rightarrow u = (2x^2 + 1)$$

b. Is there an (approximate) function/derivative pair?

Yes! $\underbrace{(2x^2 + 1)}_{\text{function}} \text{ --- } \rightarrow \underbrace{x}_{\text{deriv}}$

Let $u =$ the “function” of the function/deriv pair

$$\Rightarrow u = (2x^2 + 1)$$

c. Is the “function” of the function/deriv pair the same as the “inner” of the composite function?

(i.e., do criteria **a** and **b** suggest the same choice of u ?)

Yes! \Rightarrow u-substitution is appropriate

2. Compute du

$\begin{aligned} u &= 2x^2 + 1 \\ \Rightarrow \frac{du}{dx} &= 4x \\ \Rightarrow du &= 4x dx \\ \Rightarrow \frac{1}{4} du &= x dx \end{aligned}$

When $x = 0$, $u = 2x^2 + 1 = 2(0)^2 + 1 = 1$

When $x = 1$, $u = 2x^2 + 1 = 2(1)^2 + 1 = 3$

3. Analyze in terms of u and du

$$\int_{x=0}^{x=1} \underbrace{(2x^2 + 1)^3}_{u^3} \underbrace{x dx}_{\frac{1}{4} du} = \int_{u=1}^{u=3} u^3 \cdot \frac{1}{4} du = \frac{1}{4} \int_{u=1}^{u=3} u^3 du$$

Don't forget to re-write the limits of integration in terms of u !

4. Integrate (in terms of u).

$$\frac{1}{4} \int_{u=1}^{u=3} u^3 du = \frac{1}{4} \left[\frac{u^4}{4} \right]_{u=1}^{u=3} = \left[\frac{u^4}{16} \right]_{u=1}^{u=3} = \underbrace{\frac{(3)^4}{16}}_{F(3)} - \underbrace{\frac{(1)^4}{16}}_{F(1)} = \frac{81}{16} - \frac{1}{16} = 5$$

$\text{i.e., } \int_{x=0}^{x=1} (2x^2 + 1)^3 x dx = 5$
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