

MTH 1125 2pm Class - Test #4 - Solutions

FALL 2018

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Name _____

Show CLEARLY how you arrive at your answers!

1. **Compute:** $\int (16x^3 + 12x^2 + 8x + 4 + 2\sqrt{x}) dx =$

$$\int (16x^3 + 12x^2 + 8x + 4 + 2\sqrt{x}) dx = \int (16x^3 + 12x^2 + 8x + 4 + 2x^{\frac{1}{2}}) dx$$

$$= 16 \left[\frac{x^4}{4} \right] + 12 \left[\frac{x^3}{3} \right] + 8 \left[\frac{x^2}{2} \right] + 4x + 2 \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right) + C = 4x^4 + 4x^3 + 4x^2 + 4x + 2 \left(\frac{2}{3} \right) x^{\frac{3}{2}} + C$$

$$= 4x^4 + 4x^3 + 4x^2 + 4x + \frac{4}{3}x^{\frac{3}{2}} + C$$

i.e., $\int (16x^3 + 12x^2 + 8x + 4 + 2\sqrt{x}) dx = 4x^4 + 4x^3 + 4x^2 + 4x + \frac{4}{3}x^{\frac{3}{2}} + C$

2. **Compute:** $\int (9x^2 + 12x)^5 (3x + 2) dx =$

1. Is u-sub appropriate?

a. Is there a composite function?

Yes! $(9x^2 + 12x)^5$ (A function raised to a power is *always* a composite function!)

Let $u =$ the “inner” of the composite function

$$\Rightarrow u = (9x^2 + 12x)$$

b. Is there an (approximate) function/derivative pair?

Yes! $\underbrace{(9x^2 + 12x)}_{\text{function}} \text{ --- } \rightarrow \underbrace{(3x + 2)}_{\text{deriv}}$

Let $u =$ the “function” of the function/deriv pair

$$\Rightarrow u = (9x^2 + 12x)$$

c. Is the “function” of the function/deriv pair the same as the “inner” of the composite function?

(i.e., do criteria **a** and **b** suggest the same choice of u ?)

Yes! \Rightarrow u-substitution is appropriate

2. Compute du

$\begin{aligned} u &= 9x^2 + 12x \\ \Rightarrow \frac{du}{dx} &= 18x + 12 \\ \Rightarrow du &= (18x + 12) dx \\ \Rightarrow \frac{1}{6} du &= (3x + 2) dx \end{aligned}$
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3. Analyze in terms of u and du

$$\int \underbrace{(9x^2 + 12x)^5}_{u^5} \underbrace{(3x + 2) dx}_{\frac{1}{6} du} = \int u^5 \frac{1}{6} du = \frac{1}{6} \int u^5 du$$

4. Integrate (in terms of u).

$$\frac{1}{6} \int u^5 du = \frac{1}{6} \left[\frac{u^6}{6} \right] + C = \frac{1}{36} u^6 + C$$

5. Re-express in terms of the original variable, x .

$$\int (9x^2 + 12x)^5 (3x + 2) dx = \frac{1}{36} \underbrace{(9x^2 + 12x)^6}_{\frac{1}{36} u^6 + C} + C$$

$\text{i.e., } \int (9x^2 + 12x)^5 (3x + 2) dx = \frac{1}{36} (9x^2 + 12x)^6 + C$

3. **Compute:** $\int (5 \cos(x) + 6 \csc^2(x)) dx =$

$$\begin{aligned} \int (5 \cos(x) + 6 \csc^2(x)) dx &= 5 [\sin(x)] + 6 [-\cot(x)] + C \\ &= 5 \sin(x) - 6 \cot(x) + C \end{aligned}$$

i.e., $\int (5 \cos(x) + 6 \csc^2(x)) dx = 5 \sin(x) - 6 \cot(x) + C$

4. **Compute:** $\int \sin(6x^2 + 18x + 4)(2x + 3) dx =$

1. Is u-sub appropriate?

a. Is there a composite function?

Yes! $\sin(6x^2 + 18x + 4)$

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Let $u =$ the “inner” of the composite function

$$\Rightarrow u = (6x^2 + 18x + 4)$$

b. Is there an (approximate) function/derivative pair?

Yes! $\underbrace{(6x^2 + 18x + 4)}_{\text{function}} - - - - \rightarrow \underbrace{(2x + 3)}_{\text{deriv}}$

Let $u =$ the “function” of the function/deriv pair

$$\Rightarrow u = (3x^5 + 5x)$$

c. Is the “function” of the function/deriv pair the same as the “inner” of the composite function?

(i.e., do criteria **a** and **b** suggest the same choice of u ?)

Yes! \Rightarrow u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 6x^2 + 18x + 4 \\ \Rightarrow \frac{du}{dx} &= 12x + 18 \\ \Rightarrow du &= (12x + 18) dx \\ \Rightarrow \frac{1}{6} du &= (2x + 3) dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\sin(6x^2 + 18x + 4)}_{\sin(u)} \underbrace{(2x + 3) dx}_{\frac{1}{6} du} = \int \sin(u) \frac{1}{6} du = \frac{1}{6} \int \sin(u) du$$

4. Integrate (in terms of u).

$$\frac{1}{6} \int \sin(u) du = \frac{1}{6} [-\cos(u)] + C = -\frac{1}{6} \cos(u) + C$$

5. Re-express in terms of the original variable, x .

$$\int \sin(6x^2 + 18x + 4)(2x + 3) dx = \underbrace{-\frac{1}{6} \cos(6x^2 + 18x + 4) + C}_{-\frac{1}{6} \cos(u) + C}$$

i.e., $\int \sin(6x^2 + 18x + 4)(2x + 3) dx = -\frac{1}{6} \cos(6x^2 + 18x + 4) + C$

5. **Compute:** $\int_{-1}^1 (6x^2 + 6x + 4) dx =$

$$\begin{aligned} \int_{-1}^1 \underbrace{(6x^2 + 6x + 4)}_{f(x)} dx &= \underbrace{\left[6\frac{x^3}{3} + 6\frac{x^2}{2} + 4x \right]_{-1}^1}_{F(x)} = \underbrace{\left[2x^3 + 3x^2 + 4x \right]_{-1}^1}_{F(x)} = \\ &= \underbrace{\left[2(1)^3 + 3(1)^2 + 4(1) \right]}_{F(1)} - \underbrace{\left[2(-1)^3 + 3(-1)^2 + 4(-1) \right]}_{F(-1)} = 12 \end{aligned}$$

i.e., $\int_{-1}^1 (6x^2 + 6x + 4) dx = 12$

6. **Compute:** $\int_0^1 (4x^2 + 2)^3 x dx =$

1. Is u-sub appropriate?

a. Is there a composite function?

Yes! $(4x^2 + 2)^3$ (A function raised to a power is *always* a composite function!)

Let $u =$ the “inner” of the composite function

$$\Rightarrow u = (4x^2 + 2)$$

b. Is there an (approximate) function/derivative pair?

Yes! $\underbrace{(4x^2 + 2)}_{\text{function}} \text{ --- } \rightarrow \underbrace{x}_{\text{deriv}}$

Let $u =$ the “function” of the function/deriv pair

$$\Rightarrow u = (4x^2 + 2)$$

c. Is the “function” of the function/deriv pair the same as the “inner” of the composite function?

(i.e., do criteria **a** and **b** suggest the same choice of u ?)

Yes! \Rightarrow u-substitution is appropriate

2. Compute du

$\begin{aligned} u &= 4x^2 + 2 \\ \Rightarrow \frac{du}{dx} &= 8x \\ \Rightarrow du &= 8x dx \\ \Rightarrow \frac{1}{8} du &= x dx \end{aligned}$

When $x = 0$, $u = 4x^2 + 2 = 4(0)^2 + 2 = 2$

When $x = 1$, $u = 4x^2 + 2 = 4(1)^2 + 2 = 6$

3. Analyze in terms of u and du

$$\int_{x=0}^{x=1} \underbrace{(4x^2 + 2)^3}_{u^3} \underbrace{x dx}_{\frac{1}{8} du} = \int_{u=2}^{u=6} u^3 \cdot \frac{1}{8} du = \frac{1}{8} \int_{u=2}^{u=6} u^3 du$$

Don't forget to re-write the limits of integration in terms of u !

4. Integrate (in terms of u).

$$\frac{1}{8} \int_{u=2}^{u=6} u^3 du = \frac{1}{8} \left[\frac{u^4}{4} \right]_{u=2}^{u=6} = \left[\frac{u^4}{32} \right]_{u=2}^{u=6} = \underbrace{\frac{(6)^4}{32}}_{F(6)} - \underbrace{\frac{(2)^4}{32}}_{F(2)} = \frac{81}{2} - \frac{1}{2} = 40$$

$\text{i.e., } \int_{x=0}^{x=1} (4x^2 + 2)^3 x dx = 40$
