

MTH 1126 - Test #1 - Solutions

SPRING 2017

Pat Rossi

Name _____

Show CLEARLY how you arrive at your answers

1. Compute: $\int \frac{x^2}{(4x^3-5)^5} dx = \int (4x^3 - 5)^{-5} x^2 dx$

1. Is u -sub appropriate?

a. Is there a composite function?

Yes. $(4x^3 - 5)^{-5}$

Let $u = (4x^3 - 5)$ i.e., "Let $u =$ the 'inner' function"

b. Is there an "approximate function/derivative pair"?

Yes. $\underbrace{(4x^3 - 5)}_{\text{function}} \rightarrow \underbrace{x^2}_{\text{deriv}}$

Let $u = (4x^3 - 5)$ i.e., "Let $u =$ 'the function'"

2. Compute du

\Rightarrow	u	$=$	$4x^3 - 5$
\Rightarrow	$\frac{du}{dx}$	$=$	$12x^2$
\Rightarrow	du	$=$	$12x^2 dx$
\Rightarrow	$\frac{1}{12} du$	$=$	$x^2 dx$

3. Analyze in terms of u and du .

$$\int \underbrace{(4x^3 - 5)^{-5}}_{u^{-5}} \underbrace{x^2 dx}_{\frac{1}{12} du} = \int u^{-5} \frac{1}{12} du = \frac{1}{12} \int u^{-5} du$$

4. Integrate in terms of u

$$\frac{1}{12} \int u^{-5} du = \frac{1}{12} \frac{u^{-4}}{-4} + C = -\frac{1}{48} u^{-4} + C$$

5. Re-write in terms of x

$$\int \frac{x^2}{(4x^3-5)^5} dx = \underbrace{-\frac{1}{48} (4x^3 - 5)^{-4} + C}_{-\frac{1}{48} u^{-4} + C}$$

2. Compute: $\int \frac{x^2}{(4x^3-5)} dx = \int \frac{1}{(4x^3-5)} x^2 dx$

1. Is u -sub appropriate?

a. Is there a composite function?

Yes. $\frac{1}{(4x^3-5)} = (4x^3 - 5)^{-1}$

Let $u = (4x^3 - 5)$ i.e., “Let $u =$ the ‘inner’ function”

b. Is there an “approximate function/derivative pair”?

Yes. $\underbrace{(4x^3 - 5)}_{\text{function}} \rightarrow \underbrace{x^2}_{\text{deriv}}$

Let $u = (4x^3 - 5)$ i.e., “Let $u =$ ‘the function’”

2. Compute du

u	$=$	$4x^3 - 5$
$\Rightarrow \frac{du}{dx}$	$=$	$12x^2$
$\Rightarrow du$	$=$	$12x^2 dx$
$\Rightarrow \frac{1}{12} du$	$=$	$x^2 dx$

3. Analyze in terms of u and du .

$$\int \underbrace{\frac{1}{(4x^3 - 5)}}_{\frac{1}{u}} \underbrace{x^2 dx}_{\frac{1}{12} du} = \int \frac{1}{u} \frac{1}{12} du = \frac{1}{12} \int \frac{1}{u} du$$

4. Integrate in terms of u

$$\frac{1}{12} \int \frac{1}{u} du = \frac{1}{12} \ln |u| + C$$

5. Re-write in terms of x

$$\int \frac{x^2}{(4x^3-5)} dx = \underbrace{\frac{1}{12} \ln |4x^3 - 5| + C}_{\frac{1}{12} \ln |u| + C}$$

$$3. \text{ Compute: } \frac{d}{dx} \left[e^{(4x^5+5x^4)} \right] = \underbrace{\frac{d}{dx} \left[e^{(4x^5+5x^4)} \right]}_{\frac{d}{dx} [e^u]} = e^{(4x^5+5x^4)} \cdot \underbrace{(20x^4 + 20x^3)}_{\frac{du}{dx}} = (20x^4 + 20x^3) e^{(4x^5+5x^4)}$$

$$\boxed{\text{i.e., } \frac{d}{dx} \left[e^{(4x^5+5x^4)} \right] = (20x^4 + 20x^3) e^{(4x^5+5x^4)}}$$

$$4. \text{ Compute: } \frac{d}{dx} \left[\ln \left(\sqrt{\frac{4x^2-2x}{2x^2+1}} \right) \right] =$$

$$\frac{d}{dx} \left[\ln \left(\sqrt{\frac{4x^2-2x}{2x^2+1}} \right) \right] = \frac{d}{dx} \left[\ln \left(\left(\frac{4x^2-2x}{2x^2+1} \right)^{\frac{1}{2}} \right) \right] = \frac{d}{dx} \left[\frac{1}{2} \ln \left(\left(\frac{4x^2-2x}{2x^2+1} \right) \right) \right] = \frac{d}{dx} \left[\frac{1}{2} (\ln(4x^2 - 2x) - \ln(2x^2 + 1)) \right]$$

$$= \frac{1}{2} \left(\underbrace{\frac{1}{4x^2-2x}}_{\frac{1}{u}} \cdot \underbrace{(8x-2)}_{\frac{du}{dx}} - \underbrace{\frac{1}{2x^2+1}}_{\frac{1}{u}} \cdot \underbrace{4x}_{\frac{du}{dx}} \right) = \frac{1}{4x^2-2x} (4x-1) - \frac{1}{2x^2+1} 2x = \frac{4x-1}{4x^2-2x} - \frac{2x}{2x^2+1}$$

$$\boxed{\text{i.e., } \frac{d}{dx} \left[\ln \left(\sqrt{\frac{4x^2-2x}{2x^2+1}} \right) \right] = \frac{4x-1}{4x^2-2x} - \frac{2x}{2x^2+1}}$$

Alternatively:

$$\frac{d}{dx} \left[\ln \left(\sqrt{\frac{4x^2-2x}{2x^2+1}} \right) \right] = \frac{d}{dx} \left[\underbrace{\ln \left[\left(\frac{4x^2-2x}{2x^2+1} \right)^{\frac{1}{2}} \right]}_{\ln(u)} \right] = \frac{1}{\underbrace{\left(\frac{4x^2-2x}{2x^2+1} \right)^{\frac{1}{2}}}_{\frac{1}{u}}} \cdot \underbrace{\left(\frac{d}{dx} \left[\left(\frac{4x^2-2x}{2x^2+1} \right)^{\frac{1}{2}} \right] \right)}_{\frac{du}{dx}}$$

$$= \left(\frac{2x^2+1}{4x^2-2x} \right)^{\frac{1}{2}} \cdot \frac{1}{2} \left(\frac{4x^2-2x}{2x^2+1} \right)^{-\frac{1}{2}} \underbrace{\frac{(8x-2)(2x^2+1) - (4x)(4x^2-2x)}{(2x^2+1)^2}}_{\text{Quotient Rule}}$$

$$= \left(\frac{2x^2+1}{4x^2-2x} \right)^{\frac{1}{2}} \cdot \frac{1}{2} \left(\frac{2x^2+1}{4x^2-2x} \right)^{\frac{1}{2}} \frac{(8x-2)(2x^2+1) - (4x)(4x^2-2x)}{(2x^2+1)^2}$$

$$= \frac{1}{2} \left(\frac{2x^2+1}{4x^2-2x} \right) \frac{(8x-2)(2x^2+1) - (4x)(4x^2-2x)}{(2x^2+1)^2} = \frac{(4x-1)(2x^2+1) - (2x)(4x^2-2x)}{(4x^2-2x)(2x^2+1)}$$

$$\boxed{\text{i.e., } \frac{d}{dx} \left[\ln \left(\sqrt{\frac{4x^2-2x}{2x^2+1}} \right) \right] = \frac{(4x-1)(2x^2+1) - (2x)(4x^2-2x)}{(4x^2-2x)(2x^2+1)}}$$

5. Compute: $\int e^{(5x^2+4)}x dx =$

(a) 1. Is u -sub appropriate?

a. Is there a composite function?

Yes. $e^{(5x^2+4)}$

Let $u = 5x^2 + 4$

b. Is there an “approximate function/derivative pair”?

Yes. $\underbrace{(5x^2 + 4)}_{\text{function}} \rightarrow \underbrace{x}_{\text{deriv}}$

Let $u = 5x^2 + 4$

2. Compute du

$$\begin{array}{l} u = 5x^2 + 4 \\ \Rightarrow \frac{du}{dx} = 10x \\ \Rightarrow du = 10x dx \\ \Rightarrow \frac{1}{10} du = x dx \end{array}$$

3. Analyze in terms of u and du .

$$\int \underbrace{e^{(5x^2+4)}}_{e^u} \underbrace{x dx}_{\frac{1}{10} du} = \int e^u \frac{1}{10} du = \frac{1}{10} \int e^u du$$

4. Integrate in terms of u

$$\frac{1}{10} \int e^u du = \frac{1}{10} e^u + C$$

5. Re-write in terms of x

$$\int e^{(5x^2+4)}x dx = \underbrace{\frac{1}{10} e^{(5x^2+4)} + C}_{\frac{1}{10} e^u + C}$$

6. Compute: $\frac{d}{dx} [\operatorname{arcsec}(2x)] =$

$$\underbrace{\frac{d}{dx} [\operatorname{arcsec}(2x)]}_{\frac{d}{dx} [\operatorname{arcsec}(u)]} = \frac{1}{\underbrace{|2x| \sqrt{(2x)^2 - 1}}_{\frac{1}{|u| \sqrt{u^2 - 1}}}} \cdot \underbrace{2}_{\frac{du}{dx}} = \frac{2}{|2x| \sqrt{(2x)^2 - 1}} = \frac{1}{|x| \sqrt{4x^2 - 1}}$$

$$\boxed{\text{i.e., } \frac{d}{dx} [\operatorname{arcsec}(2x)] = \frac{2}{|2x| \sqrt{(2x)^2 - 1}} = \frac{1}{|x| \sqrt{4x^2 - 1}}}$$

7. Compute: $\int \frac{1}{\sqrt{16-4x^2}} dx = \int (16-4x^2)^{-\frac{1}{2}} dx$
↙ ↗
re-write

1 a. Is there a composite function?

Yes. $(16 - 4x^2)^{-\frac{1}{2}}$
⏟
inner

Let $u = 16 - 4x^2$

Is there an “approximate function/derivative pair”?

There does not appear to be an “approximate function/derivative pair.”

Proceeding solely on the strength of Part a, we continue, aware of the possibility that u-substitution might not work.

2. Compute du

$$\begin{array}{l} u = 16 - 4x^2 \\ \Rightarrow \frac{du}{dx} = -8x \\ \Rightarrow du = -8x dx \\ \Rightarrow -\frac{1}{8} du = x dx \end{array}$$

3. Analyze in terms of u and du .

$$\underbrace{\int (16 - 4x^2)^{-\frac{1}{2}}}_{u^{-\frac{1}{2}}} \cdot \underbrace{dx}_{-\frac{1}{8x} du} =$$

Since we cannot analyze the integral solely in terms of u and du , u-substitution alone will not work in this case.

We must try to get our integral to fit a different form. (See Next Page)

Exercise 7 Continued . . .

$$\int \frac{1}{\sqrt{16-4x^2}} dx \quad \text{compare to:} \quad \int \frac{1}{\sqrt{a^2-u^2}} du$$

If this comparison is correct, then:

$a^2 = 16$
$\Rightarrow a = 4$
$u^2 = 4x^2 = (2x)^2$
$\Rightarrow u = 2x$
$\Rightarrow \frac{du}{dx} = 2$
$\Rightarrow du = 2dx$
$\Rightarrow \frac{1}{2}du = dx$

Now analyze the integral in terms of u and du .

$$\int \underbrace{\frac{1}{\sqrt{16-4x^2}} dx}_{\frac{1}{\sqrt{a^2-u^2}} \frac{1}{2} du} = \int \frac{1}{\sqrt{a^2-u^2}} \frac{1}{2} du = \frac{1}{2} \int \frac{1}{\sqrt{a^2-u^2}} du$$

Integrate:

$$\frac{1}{2} \int \frac{1}{\sqrt{a^2-u^2}} du = \frac{1}{2} [\arcsin(\frac{u}{a})] + C = \frac{1}{2} [\arcsin(\frac{2x}{4})] + C = \frac{1}{2} \arcsin(\frac{x}{2}) + C$$

$$\boxed{\text{i.e., } \int \frac{x}{\sqrt{16-4x^2}} dx = \frac{1}{2} \arcsin(\frac{x}{2}) + C}$$

8. Compute: $\frac{d}{dx} [\arccos(x^3)] =$

$$\underbrace{\frac{d}{dx} [\arccos(x^3)]}_{\frac{d}{dx} [\arccos(u)]} = - \frac{1}{\underbrace{\sqrt{1-(x^3)^2}}_{-\frac{1}{\sqrt{1-u^2}}}} \cdot \underbrace{3x^2}_{\frac{du}{dx}} = - \frac{3x^2}{\sqrt{1-x^6}}$$

$$\boxed{\text{i.e., } \frac{d}{dx} [\arccos(x^3)] = - \frac{3x^2}{\sqrt{1-x^6}}}$$

9. Compute: $\int \frac{1}{5+16x^4} x dx =$

1 a. Is there a composite function?

Yes. $\frac{1}{5+16x^4} = \underbrace{(5 + 16x^4)^{-1}}_{\text{inner}}$

Let $u = 5 + 16x^4$

Is there an “approximate function/derivative pair”?

There does not appear to be an “approximate function/derivative pair.”

Proceeding solely on the strength of Part a, we continue, aware of the possibility that u-substitution might not work.

2. Compute du

\Rightarrow	u	$=$	$5 + 16x^4$
\Rightarrow	$\frac{du}{dx}$	$=$	$64x^3$
\Rightarrow	du	$=$	$64x^3 dx$
\Rightarrow	$\frac{1}{64} du$	$=$	$x^3 dx$

3. Analyze in terms of u and du .

$$\int \underbrace{\frac{1}{5 + 16x^4}}_{\frac{1}{u}} \cdot \underbrace{x dx}_{\frac{1}{64x^2} du} =$$

Since we cannot analyze the integral solely in terms of u and du , u-substitution alone will not work in this case.

We must try to get our integral to fit a different form. (See Next Page)

Exercise 9 Continued . . .

$$\int \frac{1}{5+16x^4} x dx \quad \text{compare to:} \quad \int \frac{1}{a^2+u^2} du$$

If this comparison is correct, then:

$a^2 = 5$
$\Rightarrow a = \sqrt{5}$
$u^2 = 16x^4 = (4x^2)^2$
$\Rightarrow u = 4x^2$
$\Rightarrow \frac{du}{dx} = 8x$
$\Rightarrow du = 8x dx$
$\Rightarrow \frac{1}{8} du = x dx$

Now analyze the integral in terms of u and du .

$$\int \underbrace{\frac{1}{5+16x^4}}_{\frac{1}{a^2+u^2}} \underbrace{x dx}_{\frac{1}{8} du} = \int \frac{1}{a^2+u^2} \frac{1}{8} du = \frac{1}{8} \int \frac{1}{a^2+u^2} du$$

Integrate:

$$\frac{1}{8} \int \frac{1}{a^2+u^2} du = \frac{1}{8} \left[\frac{1}{a} \arctan \left(\frac{u}{a} \right) \right] + C = \frac{1}{8} \left[\frac{1}{\sqrt{5}} \arctan \left(\frac{4x^2}{\sqrt{5}} \right) \right] + C = \frac{1}{8\sqrt{5}} \arctan \left(\frac{4x^2}{\sqrt{5}} \right) + C$$

i.e., $\int \frac{1}{5+16x^4} x dx = \frac{1}{8\sqrt{5}} \arctan \left(\frac{4x^2}{\sqrt{5}} \right) + C$
--

10. $z = \tan(\arccos(4x))$ Re-write this equation as an equivalent algebraic equation.

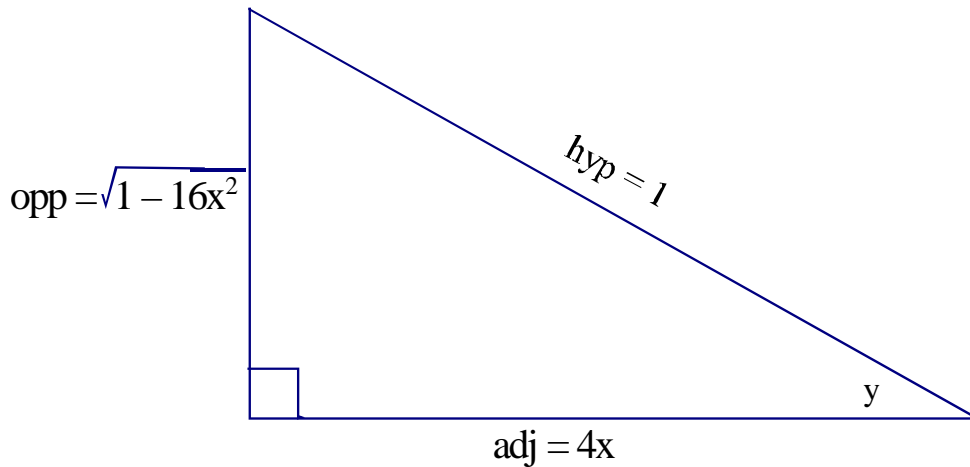
Let $y = \arccos(4x)$

Then “ y is the angle whose cosine is $4x$.”

i.e., $\cos(y) = 4x$

Draw a right triangle that depicts this relationship.

i.e., $\cos(y) = 4x = \frac{\text{adj}}{\text{hyp}} = \frac{4x}{1}$



We want $z = \tan(\arccos(4x))$

But since $y = \arccos(4x)$,

$\Rightarrow z = \tan(y)$

$\Rightarrow z = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{1-16x^2}}{4x}$

i.e., $z = \frac{\sqrt{1-16x^2}}{4x}$

Extra: Wow! 10 points (All or nothing)

Compute: $\int \frac{1}{e^{-x}+e^x} dx =$

This appears to be of the form $\int \frac{1}{u} du$, where $u = e^{-x} + e^x$.

Unfortunately, we don't find du (or a constant multiple of du) in the expression:

$$\int \underbrace{\frac{1}{e^{-x} + e^x}}_{\frac{1}{u}} \underbrace{dx}_{\substack{\text{NOT a constant} \\ \text{Multiple of } du}}$$

Therefore, we can't use u -substitution on the integral while it is in the present form.

Looking at the integral $\int \frac{1}{e^{-x}+e^x} dx$, I'd like to make it fit the form: $\int \frac{1}{a^2+u^2} du$. But, that would require us to be able to turn the e^{-x} in the denominator into a constant. We can accomplish this (without changing the value of the integrand) by multiplying both numerator and denominator by e^x .

$$\int \frac{1}{e^{-x}+e^x} dx = \int \frac{e^x}{e^x e^{-x}+e^x} dx = \int \frac{e^x}{e^x(e^{-x}+e^x)} dx = \int \frac{1}{(1+(e^x)^2)} e^x dx$$

If we let:

$$a = 1$$

$$u = e^x$$

$$du = e^x dx,$$

Then this fits (perfectly!) the desired form:

$$\int \underbrace{\frac{1}{(1+(e^x)^2)}}_{\frac{1}{a^2+u^2}} \underbrace{e^x dx}_{du} = \int \frac{1}{a^2+u^2} du = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C = \frac{1}{1} \arctan\left(\frac{e^x}{1}\right) + C = \arctan(e^x) + C$$

i.e., $\int \frac{1}{e^{-x}+e^x} dx = \arctan(e^x) + C$