MTH 2201 Test #1 - Solutions

SPRING 2016

Pat Rossi Name ____________________

Instructions

Show CLEARLY how you arrive at your answers!

1. $f(x) = 3x^4 + 4x^2 - 5x + 6 - \sqrt{x}$. Compute $f'(x)$

   Rewrite: $f(x) = 3x^4 + 4x^2 - 5x + 6 - x^{\frac{1}{2}}$

   $f'(x) = 3\left(4x^3\right) + 4(2x) - 5(1) - \frac{1}{2}x^{-\frac{1}{2}}$

   $= 12x^3 + 8x - 5 - \frac{1}{2}x^{-\frac{1}{2}}$

   $$\boxed{\text{i.e., } f'(x) = 12x^3 + 8x - 5 - \tfrac{1}{2}x^{-\frac{1}{2}}}$$

2. $f(x) = 4x^2 + 6x + 6$ Compute the slope of the line tangent to the graph of $f(x)$ at the point $(-2, 10)$.

   Slope is given by $f'(x) = 8x + 6$

   At the point $(x, y) = (-2, 10)$, $f'(x) = f'(-2) = 8(-2) + 6 = -10$.

   $$\boxed{\text{At the point } (-2,10)\text{, the slope of the line tangent to the graph of } f(x) \text{ is } -10.}$$

3. Compute: $\frac{d}{dx}\left[\left(x^2 + 2x + 1\right)\left(3x^4 - 7x^2\right)\right]$ using the Product Rule

   This is a product, so use the *product rule.*

   $$\frac{d}{dx}\left[\underbrace{\left(x^2+2x+1\right)}_{1^{\text{st}}}\underbrace{\left(3x^4-7x^2\right)}_{2^{\text{nd}}}\right] = \underbrace{\left(x^2+2x+1\right)}_{1^{\text{st}}}\underbrace{\left(12x^3-14x\right)}_{2^{\text{nd}}\text{ prime}} + \underbrace{\left(3x^4-7x^2\right)}_{2^{\text{nd}}}\underbrace{(2x+2)}_{1^{\text{st}}\text{ prime}}$$

   $$\boxed{\text{i.e., } \frac{d}{dx}\left[\left(x^2+2x+1\right)\left(3x^4-7x^2\right)\right] = \left(x^2+2x+1\right)\left(12x^3-14x\right) + \left(3x^4-7x^2\right)(2x+2)}$$

4. $f(x) = \left(8x^5 + 3x^2 + 2\right)^{15}$ Compute: $f'(x)$

   $$f'(x) = \underbrace{15\left(8x^5+3x^2+2\right)^{14}}_{\text{Power Rule As Usual}}\underbrace{\left(40x^4+6x\right)}_{\text{Deriv of inner}}$$

   $$\boxed{\text{i.e., } f'(x) = 15\left(8x^5+3x^2+2\right)^{14}\left(40x^4+6x\right)}$$

5. Compute: $\frac{d}{dx}\left[\left(\frac{8x^3+4x}{10x^3+10}\right)\right]$ using the Quotient Rule

$$\frac{d}{dx}\left[\frac{\overbrace{8x^3+4x}^{\text{top}}}{\underbrace{10x^3+10}_{\text{bottom}}}\right] = \frac{\overbrace{\left(10x^3+10\right)}^{\text{bottom}}\overbrace{\left(24x^2+4\right)}^{\text{top prime}} - \overbrace{\left(8x^3+4x\right)}^{\text{top}}\ \overbrace{\left(30x^2\right)}^{\text{bottom prime}}}{\underbrace{\left(10x^3+10\right)^2}_{\text{bottom squared}}}$$

i.e., $\frac{d}{dx}\left[\frac{8x^3+4x}{10x^3+10}\right] = \frac{\left(10x^3+10\right)\left(24x^2+4\right)-\left(8x^3+4x\right)\left(30x^2\right)}{\left(10x^3+10\right)^2}$

6. Compute: $\frac{d}{dx}\left[\left(20x^5+25x^2\right)^3\left(50x^2+20\right)^4\right] =$

In the broadest sense, this is a *product*. So use the *product rule*.

$$\frac{d}{dx}\left[\underbrace{\left(20x^5+25x^2\right)^3}_{1^{\text{st}}}\underbrace{\left(50x^2+20\right)^4}_{2^{\text{nd}}}\right] = \underbrace{\left(20x^5+25x^2\right)^3}_{1^{\text{st}}}\underbrace{\left(\frac{d}{dx}\left[\left(50x^2+20\right)^4\right]\right)}_{2^{\text{nd}}\text{ prime}}$$

$$+\underbrace{\left(50x^2+20\right)^4}_{2^{\text{nd}}}\underbrace{\left(\frac{d}{dx}\left[\left(20x^5+25x^2\right)^3\right]\right)}_{1^{\text{st}}\text{ prime}}$$

$$= \left(20x^5+25x^2\right)^3\left[\underbrace{4\left(50x^2+20\right)^3}_{\substack{\text{power rule}\\ \text{as usual}}}\cdot\underbrace{(100x)}_{\substack{\text{deriv. of}\\ \text{inner}}}\right]$$

$$+\left(50x^2+20\right)^4\left[\underbrace{3\left(20x^5+25x^2\right)^2}_{\substack{\text{power rule}\\ \text{as usual}}}\cdot\underbrace{\left(100x^4+50x\right)}_{\substack{\text{deriv. of}\\ \text{inner}}}\right]$$

i.e., $\frac{d}{dx}\left[\left(20x^5+25x^2\right)^3\left(50x^2+20\right)^4\right] = \left(20x^5+25x^2\right)^3\left[4\left(50x^2+20\right)^3\cdot(100x)\right]$

$$+\left(50x^2+20\right)^4\left[3\left(20x^5+25x^2\right)^2\cdot\left(100x^4+50x\right)\right]$$

7. Compute: $\frac{d}{dx}\left[\left(\frac{20x^5+25x^2}{50x^2+20}\right)^5\right] =$

In the broadest sense, this is a *function raised to a power*. So use the *Chain Rule*.

$$\frac{d}{dx}\left[\left(\frac{20x^5+25x^2}{50x^2+20}\right)^5\right] = \underbrace{5\left(\frac{20x^5+25x^2}{50x^2+20}\right)^4}_{\substack{\text{Power Rule}\\\text{as usual}}} \cdot \underbrace{\left(\frac{d}{dx}\left[\frac{20x^5+25x^2}{50x^2+20}\right]\right)}_{\substack{\text{deriv of}\\\text{inner function}}}$$

$$= 5\left(\frac{20x^5+25x^2}{50x^2+20}\right)^4 \cdot \frac{\left(50x^2+20\right)\left(100x^4+50x\right)-\left(20x^5+25x^2\right)(100x)}{(50x^2+20)^2}$$

i.e., $\frac{d}{dx}\left[\left(\frac{20x^5+25x^2}{50x^2+20}\right)^5\right] = 5\left(\frac{20x^5+25x^2}{50x^2+20}\right)^4 \cdot \frac{\left(50x^2+20\right)\left(100x^4+50x\right)-\left(20x^5+25x^2\right)(100x)}{(50x^2+20)^2}$