MTH 2201 - Test #2 - Solutions

Spring 2016

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Instructions. Show CLEARLY how you arrive at your answers.

1. A theme park rents out its facilities for college graduation parties, provided that at least 500 people are in the party. Park regulations prohibit parties larger than 1500. For the minimum party size of 500 people, tickets cost \$60 per person. The park offers a reduction in ticket price (to all people in the party) of \$0.03 per person for every additional person over the required minimum of 500 party-goers.

   (a) How many part-goers will maximize the park's revenue?

   (b) When the park is making maximum revenue, what will the cost per ticket be?

   (c) How many party-goers will minimize the revenue?

   1. Determine the quantity that is to be maximized.

      Maximize revenue, R.

      $R =$ (number of party-goers) (price per party-goer)

      Let n be the number of party-goers.

      Let p be the price per party-goer.

      Then $R = np$

   2. Express R as a function of *one* variable.

      Observe: $p = \$60.00-$ discount

      $$= \$60.00 - \$0.03 \underbrace{(\text{number of passengers over the minimum of 500})}_{n-100}$$

      $$= \$60.00 - \$0.03\,(n-500)$$

      $$= \$60.00 - \$0.03n + \$15.00$$

      i.e., $p = \$75.00 - \$0.03n$

      Recall: $R = np$

      $\Rightarrow R = n\underbrace{(75.00 - \$0.03n)}_{p} = \$75.00n - \$0.03n^2$

      i.e., $R\,(n) = \$75.00n - \$0.03n^2$

   3. Determine the restrictions on n.

      $500 \leq n \leq 1500$

4. Minimize $R(n)$

   Note: $R(n)$ is [1]continuous (no zero divides) on the [2]closed, [3]finite interval $[500, 1500]$.

   Hence, we can use the Absolute Max/Min Value Test.

   Find Critical numbers

   $R'(n) = \$75.00 - \$0.06n$

   a. (Type a)

      $R'(n) = \$75.00 - \$0.06n = 0$

      $\Rightarrow \$0.06n = \$75.00$

      $\Rightarrow n = \frac{\$75.00}{\$0.06} = 1250$ (crit. no.)

   b. (Type b)

      None

   Plug Critical numbers and endpoints into the *original function.*

   $R(500) = \$75.00(500) - \$0.03(500)^2 = \$30,000.00$ $\Leftarrow$ Abs. Min. Value

   $R(1250) = \$75.00(1250) - \$0.03(1250)^2 = \$46,875.00$ $\Leftarrow$Abs. Max. Value

   $R(1500) = \$75.00(1500) - \$0.03(1500)^2 = \$45,000.0$

5. Answer the original question(s)

   1250 party-goers maximize revenue.

   At maximum revenue, the cost per party-goer is:

   $\text{Cost} = \frac{\text{Revenue}}{\text{number of party-goers}} = \frac{\$46,875.00}{1250} = \$37.50$ per party-goer

   i.e., At maximum revenue, the cost per party-goer is \$37.50 per party-goer

   Revenue is minimized when there are 500 party-goers

2. $f(x) = 2x^3 - 15x^2 + 36x + 2$ on the interval $[-4, 2]$. Find the absolute maximum value and the absolute minimum value.

Note: $f(x)$ is [1]continuous (no zero divides) on the [2]closed, [3]finite interval $[-4, 2]$.

Hence, we can use the Absolute Max/Min Value Test.

(a) 1. Find Critical numbers

$f'(x) = 6x^2 - 30x + 36$

a. (Type a)

$f'(x) = 6x^2 - 30x + 36 = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow (x-2)(x-3) = 0$

$\Rightarrow x = 2$ and $x = 3$ are "type a" critical numbers

However, since $x = 3$ is outside the interval $[-4, 2]$, we discard it.

b. (Type b)

None

2. Plug Critical numbers and endpoints into the *original function.*

$f(-4) = 2(-4)^3 - 15(-4)^2 + 36(-4) + 2 = -510$ $\Leftarrow$ Abs. Min. Value

$f(2) = 2(2)^3 - 15(2)^2 + 36(2) + 2 = 30$ $\Leftarrow$Abs. Max. Value

The absolute maximum value is 30, attained at $x = 2$

The absolute minimum value is -510, attained at $x = -4$

3. $f(x) = 2x^3 + 3x^2 - 12x + 3$; determine the intervals on which $f(x)$ is increasing, determine the intervals on which $f(x)$ is decreasing, and identify the local maxes and mins.

(a) 1. Compute $f'(x)$ and find the critical numbers.

$f'(x) = 6x^2 + 6x - 12$

1. "Type a" $(f'(c) = 0)$

$\Rightarrow f'(x) = 6x^2 + 6x - 12 = 0$

$\Rightarrow 6x^2 + 6x - 12 = 0$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow (x+2)(x-1) = 0$

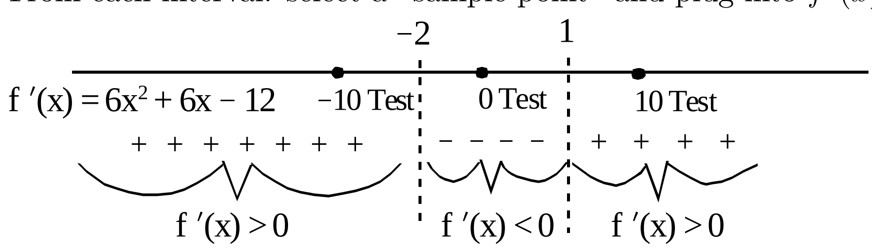
$\Rightarrow x = -2$, $x = 1$ are "type a" critical numbers

2. "Type b" $(f'(c)$ undefined $)$

There are none.

2. Draw a "sign graph" of $f'(x)$

3. From each interval. select a "sample point" and plug into $f'(x)$.



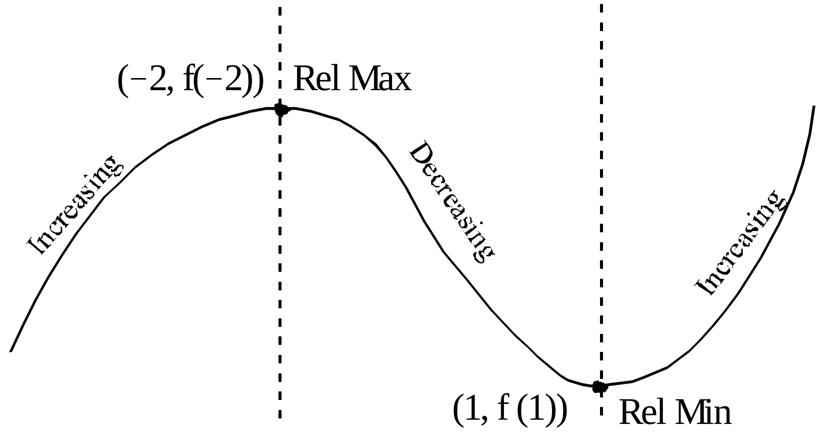
$f(x)$ is **increasing** on the intervals $(-\infty, -2)$ and $(1, \infty)$

(because $f'(x) > 0$ on these intervals).

$f(x)$ is **decreasing** on the interval $(-2, 1)$

(because $f'(x) < 0$ on this interval).

4. Sketch a rough graph of $f(x)$ to find the relative maxes and mins.



From the graph of $f(x) = 2x^3 + 3x^2 - 12x + 3$ it is clear that:

$(-2, f(-2)) = (-2, 23)$ is a **relative max.**, and

$(1, f(1)) = (1, -4)$ is a **relative min**