MTH 2201 - Test #3 - Solutions

SPRING 2016

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1. Management at a clothing manufacturer determines that in order for the production of a particular suit to be viable, the price per suit must be given by the function: $p(x) = 150 - 0.5x$, where x is the number of suits ordered by their retail outlets.

   Furthermore, the total cost of producing x suits is given by the function: $C(x) = 4000 + 0.25x^2$.

   (a) Find the *total revenue,* $R(x)$

   Revenue $= R(x) =$ (price per suit) $\cdot$ (the number of suits)

   $\Rightarrow R(x) = (150 - 0.5x)\,x = 150x - 0.5x^2$

   Total Revenue $= R(x) = 150x - 0.5x^2$

   (b) Find the *total profit,* $P(x)$

   Profit $P(x) =$ Revenue $-$ Cost $= R(x) - C(x)$

   $\Rightarrow P(x) = \left(150x - 0.5x^2\right) - \left(4000 + 0.25x^2\right) = -0.75x^2 + 150x - 4000$

   Total Profit $= P(x) = -0.75x^2 + 150x - 4000$

   (c) How many suits must the manufacturer produce and distribute to their retail stores in order maximize the profit?

   Method 1: Since maximum profit occurs when Marginal Revenue is equal to Marginal Cost, set $R'(x) = C'(x)$, and solve for x.

   $R'(x) = 150 - x$

   $C'(x) = 0.5x$

   $R'(x) = C'(x)$

   $\Rightarrow 150 - x = 0.5x$

   $\Rightarrow 150 = 1.5x$

   $\Rightarrow 100 = x$

   i.e., Marginal Revenue is equal to Marginal Cost when $x = 100$.

   Hence, Profit is maximized when production/sales is at the level of 100 suits.

Method 2: Maximize $P(x)$ by finding the critical number(s) and graphing $P(x)$.

i) $P'(x) = -1.5x + 150$

a) "Type a" $(P'(c) = 0)$

$P'(x) = -1.5x + 150 = 0$

$1.5x = 150$

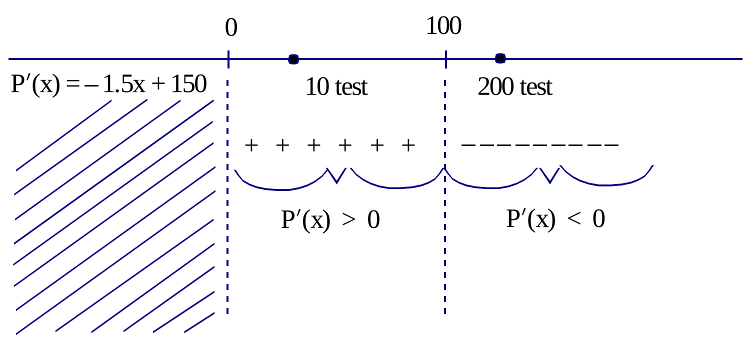
$\Rightarrow x = 100$ critical number

b) "Type b" ($P'(c)$ undefined)

None

ii) Draw a sign graph of $P'(x)$

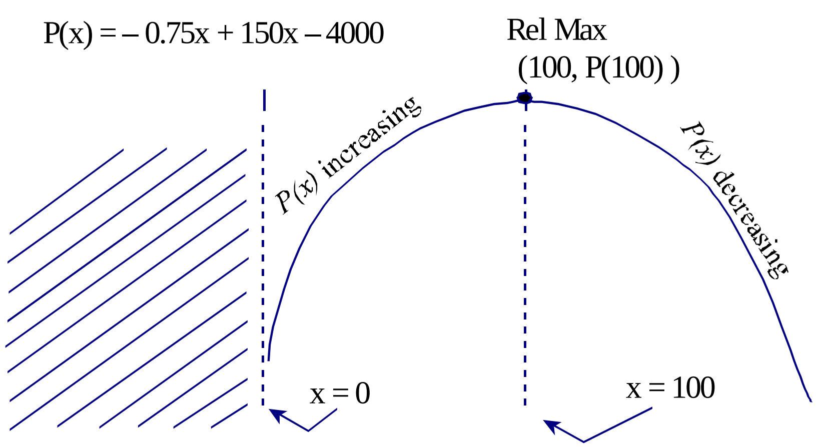
iii) Pick test points



$P(x)$ is **increasing** on the interval $(0, 100)$

$P(x)$ is **decreasing** on the interval $(100, \infty)$

iv) Sketch a rough graph of $P(x)$



From the graph, we see that the abs max value of $P(x)$ is attained at $x = 100$.

$P(100) = -0.75(100)^2 + 150(100) - 4000 = \$3500.00$ Abs Max Value

Production of $x = 100$ units maximizes profit

2. A manufacturer finds that in producing q units per day, (for $0 < q < 15,000$) three different kinds of cost are involved.

   - A fixed cost of \$2000 per day
   - A production cost of \$6.25 per day for each unit produced
   - An ordering cost of $\$2000q^{-1}$ per day (i.e. $\$\frac{2000}{q}$ per day).

   (a) Express the total cost as a function of q

   Let C_F, C_P, C_O be the (total) Fixed Cost, (total) Production Cost, and (total) Ordering Cost, respectivley. Then

   $C_F = \$2000$

   $C_O = \$2000q^{-1}$

   But we don't know C_P

   Observe: $C_P = (\text{price per item})\,(\#\text{ items produced})$

   $\Rightarrow C_P = (\$6.25)\,q = \$6.25q$

   Next Observe: Total Cost, $C\,(q) = C_F + C_P + C_O = \$2000 + \$6.25q + \$2000q^{-1}$

   i.e., $C\,(q) = \$2000 + \$6.25q + \$2000q^{-1}$

   (b) Determine the level of production that minimizes the total cost.

   i) Compute $C'\,(q)$ and find critical numbers

   $C'\,(q) = \$6.25 - \$2000q^{-2} = \$6.25 - \frac{\$2000}{q^2}$

   i) **"Type a"** $(C'\,(q) = 0)$

   Set $C'\,(q) = \$6.25 - \frac{\$2000}{q^2} = 0$

   $\Rightarrow \$6.25 - \frac{\$2000}{q^2} = 0$

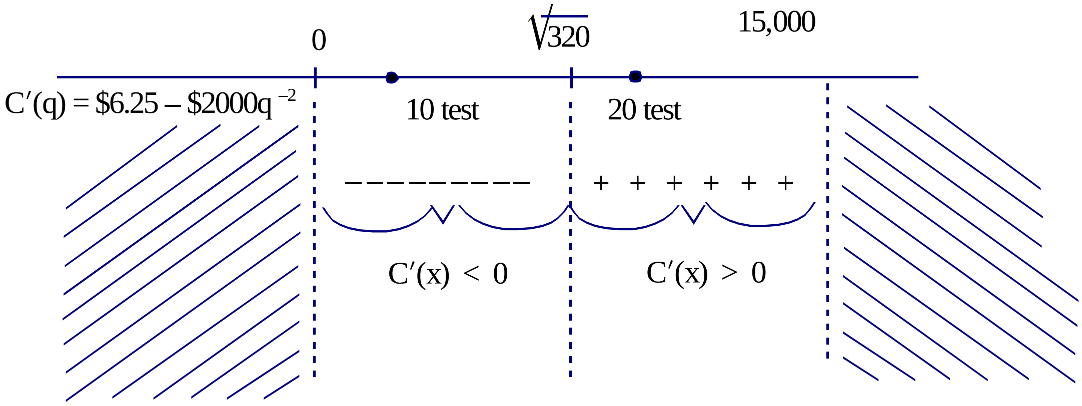
   $\Rightarrow \$6.25 = \frac{\$2000}{q^2}$

   $\Rightarrow q^2 = \frac{\$2000}{\$6.25} = 320$

   $\Rightarrow q = \sqrt{320} = 17.89$

   ii) Draw a sign graph of $P'\,(x)$

   iii) Pick test points

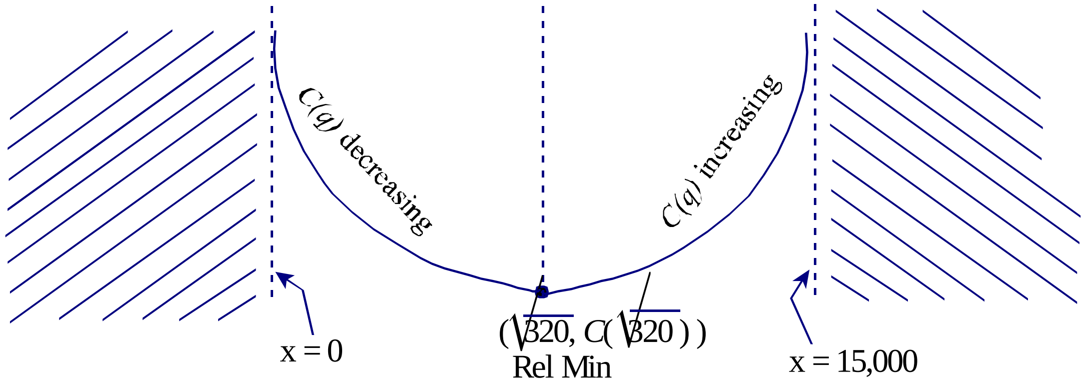


$C(x)$ is **decreasing** on the interval $\left(0, \sqrt{320}\right)$

$C(x)$ is **increasing** on the interval $\left(\sqrt{320}, 15,000\right)$

iv) Sketch a rough graph of $P(x)$

$$C(q) = \$2000 + \$6.25q + \$2000q^{-1}$$



From the graph, $C(q)$ attains its absolute minimum value at $q = \sqrt{320} \approx 17.89$

OOPS! q is the number of *units* produced daily. (i.e., must be a whole number.)

So we will test the whole number values of q that are closest to 17.89

$C(17) = 2000 + 6.25(17) + 2000(17)^{-1} = 2223.90$

$C(18) = 2000 + 6.25(18) + 2000(18)^{-1} = 2223.61$ Abs Min Value

18 units is the level of production that minimizes the total cost.

3. Management at a computer manufacturer finds that *daily profit* (in dollars) from the production and sale of its laptop computer is given by the function: $P(x) = -0.004x^3 - 0.3x^2 + 600x - 800$.

Currently, the manufacturer produces 9 laptops daily.

(a) What is the current daily profit?

@ current level of production of $x = 9$ laptops per day, the profit is given by:

$P(9) = -0.004(9)^3 - 0.3(9)^2 + 600(9) - 800 = \$4572.78$

Daily Profit $= P(9) = \$4572.78$

(b) What is the marginal profit at the current level of production?

Marginal Profit $= P'(x) = -0.012\,x^2 - 0.6x + 600$

@ current level of production of $x = 9$ laptops per day, the marginal profit is given by:

$P'(9) = -0.012(9)^2 - 0.6(9) + 600 = \$593.63$

Marginal Profit $= P'(9) = \$593.63$

(c) Estimate the approximate *increase in profit* associated with an increase in production of one unit per day, from 9 units to 10 units.

The approximate increase of profit associated with an *increase in production of one unit* from 9 to 10 laptops per day IS the marginal profit, evaluated at $x = 9$.

Approximate increase in profit $=$ \$593.63