MTH 2201 Test #4 - Solutions

Spring 2016

Pat Rossi   Name ____________________

Instructions: Show CLEARLY how you arrive at your answers.

1. Given that $\ln(2) \approx 0.7$ and $\ln(6) \approx 1.8$, Approximate the following, without the use of a calculator:

   (a) $\ln(12) = \ln(2 \cdot 6) = \ln(2) + \ln(6) \approx 0.7 + 1.8 = 2.5$

   (b) $\ln(3) = \ln\left(\frac{6}{2}\right) = \ln(6) - \ln(2) \approx 1.8 = 0.7 = 1.1$

   (c) $\ln(8) = \ln\left(2^3\right) = 3\ln(2) \approx 3(0.7) = 2.1$

2. Compute: $\frac{d}{dx}\left[\ln\left(3x^2+4x\right)\right] =$

$$\frac{d}{dx}\left[\underbrace{\ln\left(3x^2+4x\right)}_{\ln(g(x))}\right] = \underbrace{\frac{1}{3x^2+4x}}_{\frac{1}{g(x)}} \cdot \underbrace{(6x+4)}_{g'(x)}$$

i.e., $\frac{d}{dx}\left[\ln\left(3x^2+4x\right)\right] = \frac{1}{3x^2+4x} \cdot (6x+4)$

3. Compute: $\frac{d}{dx}\left[\ln\left(\frac{2x^3+8x}{4x^3+4}\right)\right] =$

$$\frac{d}{dx}\left[\ln\left(\frac{2x^3+8x}{4x^3+4}\right)\right] = \frac{d}{dx}\left[\ln\left(2x^3+8x\right) - \ln\left(4x^3+4\right)\right] = \frac{1}{2x^3+8x}\left(6x^2+8\right) - \frac{1}{4x^3+4}\left(12x^2\right)$$

i.e., $\frac{d}{dx}\left[\ln\left(\frac{2x^3+8x}{4x^3+4}\right)\right] = \frac{1}{2x^3+8x}\left(6x^2+8\right) - \frac{1}{4x^3+4}\left(12x^2\right)$

4. Compute: $\frac{d}{dx}\left[e^{4x^3}\right] =$

$$\frac{d}{dx}\left[\underbrace{e^{4x^3}}_{e^{g(x)}}\right] = \underbrace{e^{4x^3}}_{e^{g(x)}} \cdot \underbrace{12x^2}_{g'(x)}$$

i.e., $\frac{d}{dx}\left[e^{4x^3}\right] = e^{4x^3} \cdot 12x^2$

5. Compute: $\int\left(6x^2+6x+4\right)dx =$

$$\int\left(6x^2+6x+4\right)dx = 6\left[\frac{x^3}{3}\right] + 6\left[\frac{x^2}{2}\right] + 4x + C = 2x^3 + 3x^2 + 4x + C$$

i.e., $\int\left(6x^2+6x+4\right)dx = 2x^3+3x^2+4x+C$

6. Compute: $\int \frac{5}{x} dx =$

$\int \frac{5}{x} dx = \int 5 \frac{1}{x} dx = 5 \int \frac{1}{x} dx = 5 \left[\ln(x)\right] + C = 5 \ln(x) + C$

i.e., $\int \frac{5}{x} dx = 5 \ln(x) + C$

7. Compute: $\int e^{7x} dx =$

$$\int \underbrace{e^{7x}}_{e^{kx}} dx = \underbrace{\frac{1}{7} e^{7x}}_{\frac{1}{k} e^{kx}} + C$$

i.e., $\int e^{7x} dx = \frac{1}{7} e^{7x} + C$

8. Compute: $\int_{x=-1}^{x=2} \left(3x^2 - 4x + 4\right) dx =$

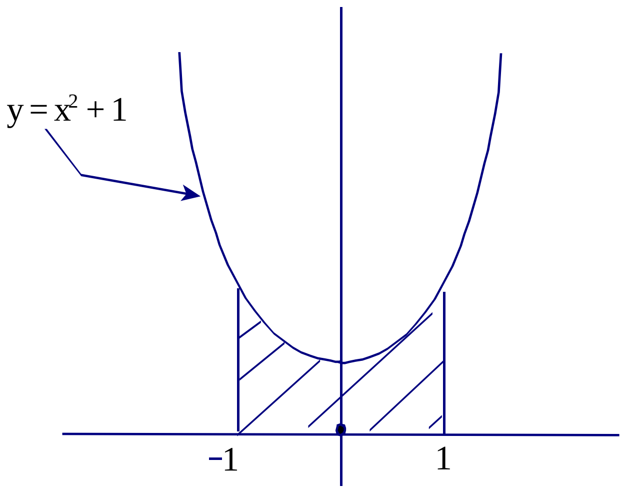
$$\int_{x=-1}^{x=2} \left(3x^2 - 4x + 4\right) dx = \underbrace{\left[3\left[\frac{x^3}{3}\right] - 4\left[\frac{x^2}{2}\right] + 4x\right]_{-1}^{2}}_{F(x)} = \underbrace{\left[x^3 - 2x^2 + 4x\right]_{-1}^{2}}_{F(x)}$$

$$= \underbrace{\left[(2)^3 - 2(2)^2 + 4(2)\right]}_{F(2)} - \underbrace{\left[(-1)^3 - 2(-1)^2 + 4(-1)\right]}_{F(-1)} = 8 - (-7) = 15$$

i.e., $\int_{x=-1}^{x=2} \left(3x^2 - 4x + 4\right) dx = 15$

9. Compute the area bounded by the graph of $f(x) = x^2 + 1$ and the x-axis between $x = -1$ and $x = 1$

First, graph the function.



Note: Between $x = -1$ and $x = 1$, the graph of $f(x)$ lies above the x-axis. (i.e., $f(x) \geq 0$)

Hence, the area is given by:

$$\text{Area} = \int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} \left(x^2+1\right) dx = \underbrace{\left[\frac{x^3}{3} + x\right]_{-1}^{1}}_{F(x)} = \underbrace{\left[\frac{(1)^3}{3} + (1)\right]}_{F(1)} - \underbrace{\left[\frac{(-1)^3}{3} + (-1)\right]}_{F(-1)}$$

$$= \left[\tfrac{1}{3} + 1\right] - \left[-\tfrac{1}{3} - 1\right] = \tfrac{8}{3}$$

i.e., Area $= \frac{8}{3}$

10. Given the demand function $q = D(x) = \frac{100}{(x+3)^2}$ and the current price of $x = 1$:

(a) Compute the elasticity function, $E(x)$

$E(x) = -\frac{xD'(x)}{D(x)}$

$D(x) = \frac{100}{(x+3)^2} = 100(x+3)^{-2}$

i.e., $D(x) = 100(x+3)^{-2}$

$D'(x) = -200(x+3)^{-3}(1) = -200(x+3)^{-3} = -\frac{200}{(x+3)^3}$

i.e., $D'(x) = -\frac{200}{(x+3)^3}$

$E(x) = -\frac{xD'(x)}{D(x)} = -\frac{x\left(-\frac{200}{(x+3)^3}\right)}{\left(\frac{100}{(x+3)^2}\right)} = x\left(\frac{200}{(x+3)^3}\right)\left(\frac{(x+3)^2}{100}\right) = \frac{2x}{x+3}$

i.e., $E(x) = \frac{2x}{x+3}$

(b) Compute the value of the elasticity function at the current price

At $x = 1$, $\quad E(x) = E(1) = \frac{2(1)}{(1)+3} = \frac{1}{2}$

i.e., $E(1) = \frac{1}{2}$

(c) Interpret the result

Since $E(1) < 1$, the elasticity of demand is *inelastic.*
This means that a unit increase in price will NOT result in a significant decrease in demand and consequently, a unit increase in price will result in an increase in revenue.

(d) Determine the price x at which the revenue is maximized

Revenue is maximized when $E(x) = 1$

To find this value of x, we set $E(x) = 1$, and solve for x.

$E(x) = \frac{2x}{x+3} = 1$

i.e., $\frac{2x}{x+3} = 1$

$\Rightarrow 2x = x + 3$

$\Rightarrow x = 3$

i.e., Total Revenue is maxmized when $x = 3$