Logic Homework Exercises #3 (Quantifiers) - Solutions

Summer 2017

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Instructions. Negate the following statements:

1. All grapefruit are pink.

   Negation: There exists at least one grapefruit that is not pink

   or: There exists a grapefruit that is not pink

   or: There exists one grapefruit that is not pink

   or: Some grapefruit are not pink.

2. Some celebrities are modest.

   Negation: No celebrities are modest.

3. No one ever lost money by underestimating the intelligence of the American public.

   Negation: There exists at least one person who lost money by underestimating the intelligence of the American public.

   or: There exists one person who lost money by underestimating the intelligence of the American public.

   or: There exists a person who lost money by underestimating the intelligence of the American public.

   or: Some people have lost money by underestimating the intelligence of the American public.

4. Some people are more than ten feet tall.

   Negation: There does not exist a person who is more than ten feet tall.

   or: No one is more than ten feet tall.

5. No one weighs more than two thousand pounds.

   Negation: There exists at least one person who weighs more than two thousand pounds.

   or: There exists one person who weighs more than two thousand pounds.

or: There exists a person who weighs more than two thousand pounds.

or: There exist some people who weigh more than two thousand pounds.

or: Some people weigh more than two thousand pounds.

6. All snakes are poisonous.

   Negation: There exists at least one snake that is not poisonous.

   or: There exists one snake that is not poisonous.

   or: There exists a snake that is not poisonous.

   or: Some snakes are not poisonous.

7. Some whales can stay under water for two days without surfacing for air.

   Negation: There does not exist a whale that can stay under water for two days without surfacing for air.

   or: No whale can stay under water for two days without surfacing for air.

8. $\exists x \exists y,\ p(x, y)$

   Negation: $\forall x \forall y,\ \neg p(x, y)$

9. $\exists x \forall y \exists z,\ p(x, y, z)$

   Negation: $\forall x \exists y \forall z,\ \neg p(x, y, z)$

10. $\forall x \forall y \exists z,\ p(x, y, z)$

    Negation: $\exists x \exists y \forall z,\ \neg p(x, y, z)$

11. $\forall$ real numbers x, $\exists$ a real number y, such that $y = \sqrt{x}$.

    (i.e. For all real numbers x, there exists a real number y, such that $y = \sqrt{x}$.)

    Negation: $\exists$ a real number x, $\forall$ real numbers y, $y \neq \sqrt{x}$.

    (i.e. There exists a real number x, such that for all real numbers y, $y \neq \sqrt{x}$.)

12. $\exists$ a real number x, such that $\forall$ real numbers y, $x \neq \sin(y)$.

    (i.e. There exists a real number x, such that for all real numbers y, $x \neq \sin(y)$.)

    Negation: $\forall$ real numbers x, $\exists$ a real number y, such that $x = \sin(y)$.

(i.e. For all real numbers x, there exists a real number y, such that $x = \sin(y)$.)

13. $\exists$ a real number z, such that $\forall$ integers x and y, $z \neq \frac{x}{y}$.

    (i.e., There exists a real number z, such that for all integers x and y, $z \neq \frac{x}{y}$.)

    Negation: $\forall$ real numbers z, $\exists$ integers x and y, such that $z = \frac{x}{y}$.

    (i.e., For all real numbers z, There exist integers x and y, such that $z = \frac{x}{y}$.)

14. $\forall$ real numbers x, $\forall$ non-zero real numbers y, $\exists$ a real number z, such that $z = \frac{x}{y}$.

    (i.e., For all real numbers x, and for all non-zero real numbers y, there exists a real number z, such that $z = \frac{x}{y}$.)

    Negation: $\exists$ a real number x, and $\exists$ a non-zero real number y, such that $\forall$ real numbers z, $z \neq \frac{x}{y}$.

    (i.e., There exists a real number x, and a non-zero real number y, such that for all real numbers z, $z \neq \frac{x}{y}$.)

    Disprove the following statements by providing a suitable counter-example:

15. If n is prime, then $2n + 1$ is also prime.

    Counter-example: There are numerous counter-examples. $n = 7$ yields $2n+1 = 15$, which is not prime.

    Other counter-examples include: $n = 13$, $n = 17$; $n = 19$; etc.

16. All birds can fly.

    Counter-example: The penguin is a bird that cannot fly.

    Other counter-examples: Perhaps some who are well-versed in zoology may have other counter-examples.)

17. All months have at least 30 days.

    Counter-example: The month of February never has more than 29 days.