

# Logic Homework Exercises #4 (Arguments) - Solutions

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**Instructions.** Determine whether each argument is valid.

1. If I study hard, then I will get A's. I will study hard. Therefore, I will get A's.

**Solution:** If we make the assignment:

$p$  : I study hard.     $r$  : I will get A's.

The argument is of the form:

If  $\underbrace{\text{I study hard}}_p$ ,  $\underbrace{\text{then}}_{\rightarrow}$   $\underbrace{\text{I will get A's}}_r$ .  $\underbrace{\text{I will study hard}}_p$ . Therefore,  $\underbrace{\text{I will get A's}}_r$ .

Our premises are:

$p_1 : p \rightarrow r$

$p_2 : p$

Our conclusion is:

$q : r$

So our argument has the form:  $(p_1 \wedge p_2) \rightarrow q$

$p$	$r$	$p_1 : p \rightarrow r$	$p_2 : p$	$q : r$	$p_1 \wedge p_2$	$(p_1 \wedge p_2) \rightarrow q$
T	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	F	F	F	T

Since the argument is a tautology, the argument is VALID.

2. If I study hard, then I get A's. If I don't get rich, then I don't get A's. Therefore, I get rich.

**Solution:** If we make the assignment:

$p$  : I study hard.     $r$  : I get A's.     $s$  : I get rich

The argument is of the form:

If I study hard, then I get A's. If I don't get rich, then I don't get A's. Therefore, I get rich.

$\underbrace{\hspace{1.5cm}}_p \quad \underbrace{\hspace{0.5cm}}_{\rightarrow} \quad \underbrace{\hspace{1.5cm}}_r \quad \underbrace{\hspace{1.5cm}}_{\neg s} \quad \underbrace{\hspace{0.5cm}}_{\rightarrow} \quad \underbrace{\hspace{1.5cm}}_{\neg r} \quad \underbrace{\hspace{1.5cm}}_s$

Our premises are:

$p_1 : p \rightarrow r$

$p_2 : \neg s \rightarrow \neg r$

Our conclusion is:

$q : s$

So our argument has the form:  $(p_1 \wedge p_2) \rightarrow q$

$p$	$r$	$s$	$\neg r$	$\neg s$	$p_1 : p \rightarrow r$	$p_2 : \neg s \rightarrow \neg r$	$q : s$	$(p_1 \wedge p_2)$	$(p_1 \wedge p_2) \rightarrow q$
T	T	T	F	F	T	T	T	T	T
T	T	F	F	T	T	F	F	F	T
T	F	T	T	F	F	T	T	F	T
T	F	F	T	T	F	T	F	F	T
F	T	T	F	F	T	T	T	T	T
F	T	F	F	T	T	F	F	F	T
F	F	T	T	F	T	T	T	T	T
F	F	F	T	T	T	T	F	T	F

Since the argument is not a tautology, the argument is INVALID.

3. I study hard, if and only if I get rich. I get rich. Therefore, I study hard.

**Solution:** If we make the assignment:

$p$  : I study hard.     $r$  : I get rich,

The argument is of the form:

$\underbrace{\text{I study hard}}_p$   $\underbrace{\text{if and only if}}_{\leftrightarrow}$   $\underbrace{\text{I get rich.}}_r$   $\underbrace{\text{I get rich}}_r$  Therefore,  $\underbrace{\text{I study hard.}}_p$

Our premises are:

$p_1 : p \leftrightarrow r$

$p_2 : r$

Our conclusion is:

$q : p$

So our argument has the form:  $(p_1 \wedge p_2) \rightarrow q$

$p$	$r$	$p_1 : p \leftrightarrow r$	$p_2 : r$	$q : p$	$(p_1 \wedge p_2)$	$(p_1 \wedge p_2) \rightarrow q$
T	T	T	T	T	T	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	T	F	F	F	T

Since the argument is a tautology, the argument is VALID.

4. If I study hard or I get rich, then I get A's. I get A's. Therefore, if I don't study hard, then I get rich.

**Solution:** If we make the assignment:

$p$  : I study hard.     $r$  : I get rich     $s$  : I get A's

The argument is of the form:

If  $\underbrace{\text{I study hard}}_p$   $\vee$   $\underbrace{\text{I get rich}}_r$ , then  $\underbrace{\text{I get A's}}_s$ .  $\underbrace{\text{I get A's}}_s$ .

Therefore, if  $\underbrace{\text{I don't study hard}}_{\neg p}$ , then  $\underbrace{\text{I get rich}}_r$ .

Our premises are:

$p_1 : (p \vee r) \rightarrow s$

$p_2 : s$

Our conclusion is:

$q : \neg p \rightarrow r$

So our argument has the form:  $(p_1 \wedge p_2) \rightarrow q$

$p$	$r$	$s$	$\neg p$	$p \vee r$	$p_1 : (p \vee r) \rightarrow s$	$p_2 : s$	$p_1 \wedge p_2$	$q : \neg p \rightarrow r$	$(p_1 \wedge p_2) \rightarrow q$
T	T	T	F	T	T	T	T	T	T
T	T	F	F	T	F	F	F	T	T
T	F	T	F	T	T	T	T	T	T
T	F	F	F	T	F	F	F	T	T
F	T	T	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F	T	T
F	F	T	T	F	T	T	T	F	F
F	F	F	T	F	T	F	F	F	T

Since the argument is not a tautology, the argument is INVALID.

5. If I study hard, then I get A's or I get rich. I don't get A's and I don't get rich. Therefore, I don't study hard.

**Solution:** If we make the assignment:

$p$  : I study hard.     $r$  : I get A's     $s$  : I get rich

The argument is of the form:

If I study hard, then (I get A's or I get rich.) I don't get A's and I don't get rich.

$\underbrace{\hspace{1.5cm}}_p \quad \rightarrow \quad \underbrace{\hspace{1.5cm}}_r \quad \vee \quad \underbrace{\hspace{1.5cm}}_s \quad \underbrace{\hspace{1.5cm}}_{\neg r} \quad \wedge \quad \underbrace{\hspace{1.5cm}}_{\neg s}$

Therefore, I don't study hard.

$\underbrace{\hspace{3cm}}_{\neg p}$

Our premises are:

$p_1 : p \rightarrow (r \vee s)$

$p_2 : \neg r \wedge \neg s$

Our conclusion is:

$q : \neg p$

So our argument has the form:  $(p_1 \wedge p_2) \rightarrow q$

$p$	$r$	$s$	$\neg p$	$\neg r$	$\neg s$	$(r \vee s)$	$p_1 : p \rightarrow (r \vee s)$	$p_2 : \neg r \wedge \neg s$	$p_1 \wedge p_2$	$q : \neg p$	$(p_1 \wedge p_2) \rightarrow q$
T	T	T	F	F	F	T	T	F	F	F	T
T	T	F	F	F	T	T	T	F	F	F	T
T	F	T	F	T	F	T	T	F	F	F	T
T	F	F	F	T	T	F	F	T	F	F	T
F	T	T	T	F	F	T	T	F	F	T	T
F	T	F	T	F	T	T	T	F	F	T	T
F	F	T	T	T	F	T	T	F	F	T	T
F	F	F	T	T	T	F	T	T	T	T	T

Since the argument is a tautology, the argument is VALID.