

Logic Exercise Set #6 - Solutions

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Name _____

Instructions. Determine whether the given arguments are valid.

1. $(p \rightarrow q) \wedge (q \rightarrow r) \therefore (p \rightarrow r)$

Answer:

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Since the argument is a tautology, it is **VALID**.

2. $[(p \rightarrow q) \wedge \sim p] \therefore \sim q$

ANSWER:

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$(p \rightarrow q) \wedge \sim p$	$[(p \rightarrow q) \wedge \sim p] \rightarrow (\sim q)$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	T	T	T	T

Since the argument is not a tautology, it is **INVALID**

3. $[(p \rightarrow q) \wedge q] \therefore p$

ANSWER:

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge q$	$[(p \rightarrow q) \wedge q] \rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

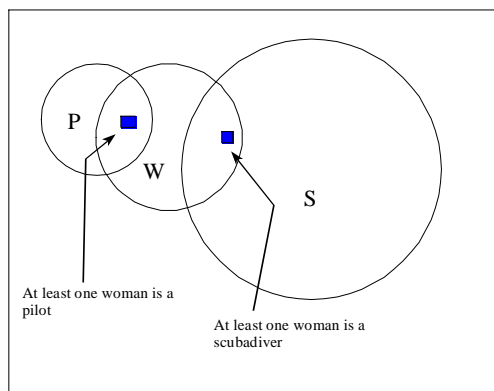
Since the argument is not a tautology, it is **INVALID**

4. No pilots are scubadivers. Some women are scubadivers. Therefore, no women are pilots.

Answer: Making the following assignments:

P - pilots, **S** - scubadivers, **W** - women

We have:



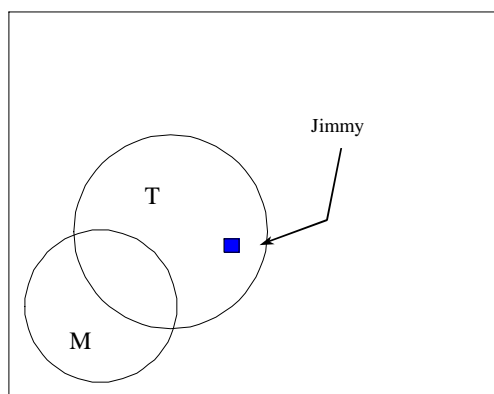
Since the hypotheses can be represented in such a way that the conclusion is false, the argument is **INVALID**.

5. Some musicians are tone deaf. Jimmy is tone deaf. Therefore, Jimmy is a musician.

Answer: Making the following assignments:

M - musicians, **T** - tone deaf

We have:



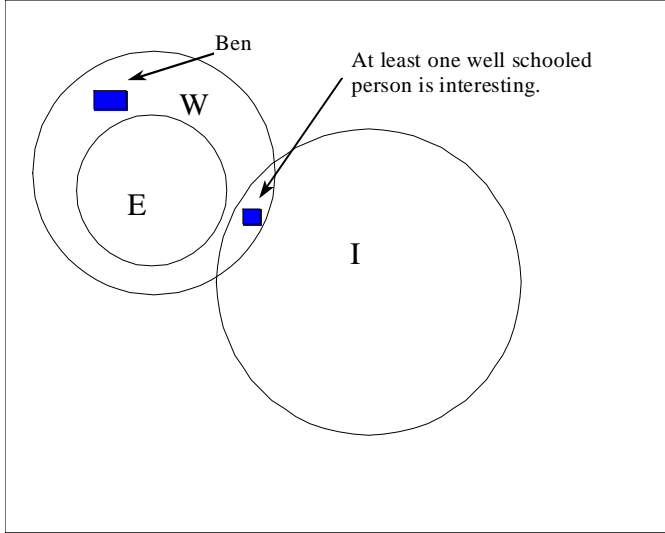
Since the hypotheses can be represented in such a way that the conclusion is false, the argument is **INVALID**.

6. All Europeans are well schooled. Some people who are well schooled are interesting. Ben is not interesting. Therefore, Ben is not European.

Answer: Making the following assignments:

E - Europeans, **W** - people who are well schooled, **I** - interesting people

We have:



Since the hypotheses can be represented in such a way that the conclusion is false, the argument is **INVALID**.

7. Give the converse and the contrapositive of the following statements:

- (a) If $x + y = 1$, then $x^2 + y^2 \geq 1$.

Converse: If $x^2 + y^2 \geq 1$, then $x + y = 1$.

Contrapositive: If $x^2 + y^2 < 1$, then $x + y \neq 1$.

- (b) If $2 + 2 = 4$, then $3 + 3 = 8$.

Converse: If $3 + 3 = 8$, then $2 + 2 = 4$.

Contrapositive: If $3 + 3 \neq 8$, then $2 + 2 \neq 4$.

- (c) If $x > 0$, then $x^2 > 0$.

Converse: If $x^2 > 0$, then $x > 0$.

Contrapositive: If $x^2 \leq 0$, then $x \leq 0$.

- (d) If my car is not running, then I will walk to school.

Converse: If I walk to school, then my car is not running.

Contrapositive: If I don't walk to school, then my car is running.

- (e) If I have any money left over, I will pay the rent.

Converse: If I pay the rent, then I have money left over.

Contrapositive: If I don't pay the rent, then I don't have money left over.