

## Logic Homework Exercises #3 (Quantifiers)

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**Instructions.** Negate the following statements:

1. All grapefruit are pink.
2. Some celebrities are modest.
3. No one ever lost money by underestimating the intelligence of the American public.
4. Some people are more than ten feet tall.
5. No one weighs more than two thousand pounds.
6. All snakes are poisonous.
7. Some whales can stay under water for two days without surfacing for air.
8.  $\exists x \exists y, p(x, y)$
9.  $\exists x \forall y \exists z, p(x, y, z)$
10.  $\forall x \forall y \exists z, p(x, y, z)$
11.  $\forall$  real numbers  $x$ ,  $\exists$  a real number  $y$ , such that  $y = \sqrt{x}$ .  
(i.e. For all real numbers  $x$ , there exists a real number  $y$ , such that  $y = \sqrt{x}$ .)
12.  $\exists$  a real number  $x$ , such that  $\forall$  real numbers  $y$ ,  $x \neq \sin(y)$ .  
(i.e. There exists a real number  $x$ , such that for all real numbers  $y$ ,  $x \neq \sin(y)$ .)
13.  $\exists$  a real number  $z$ , such that  $\forall$  integers  $x$  and  $y$ ,  $z \neq \frac{x}{y}$ .  
(i.e., There exists a real number  $z$ , such that for all real numbers  $x$  and  $y$ ,  $z \neq \frac{x}{y}$ .)
14.  $\forall$  real numbers  $x$ ,  $\forall$  non-zero real numbers  $y$ ,  $\exists$  a real number  $z$ , such that  $z = \frac{x}{y}$ .  
(i.e., For all real numbers  $x$ , and for all non-zero real numbers  $y$ , there exists a real number  $z$ , such that  $z = \frac{x}{y}$ .)

**Disprove the following statements by providing a suitable counter-example:**

15. If  $n$  is prime, then  $2n + 1$  is also prime.

16. All birds can fly.

17. All months have at least 30 days.