

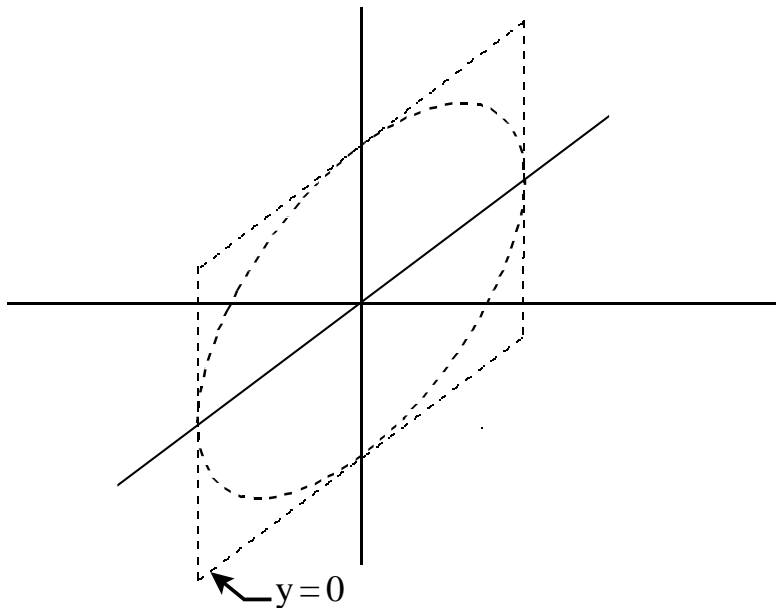
13.  $x^2 + \frac{y^2}{4} + z^2 = 1$

For any fixed value of  $y$  between  $-2$  and  $2$ , the graph is a circle.

For example:

For  $y = 0$ ; the equation becomes  $x^2 + z^2 = 1$

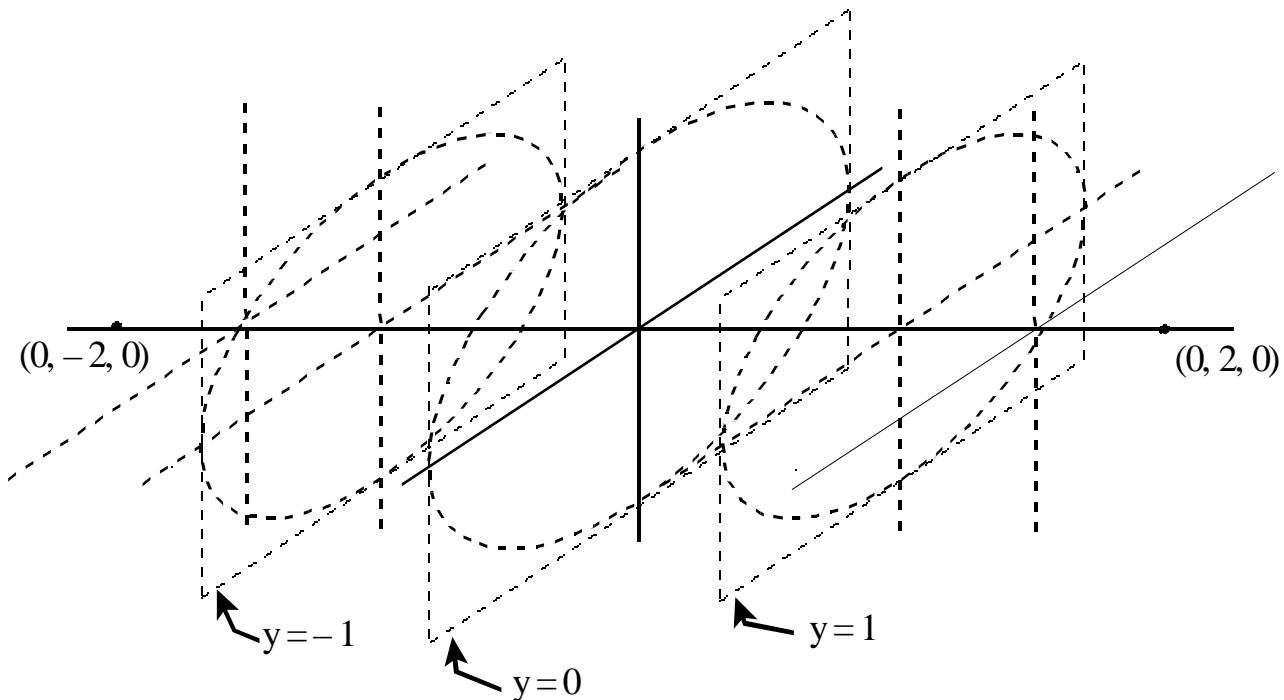
This is a circle of radius 1, with center at  $(x, z) = (0, 0)$



For  $y = \pm 1$ ; the equation becomes  $x^2 + \frac{1}{4} + z^2 = 1 \Rightarrow x^2 + z^2 = \frac{3}{4} \Rightarrow x^2 + z^2 = \left(\frac{\sqrt{3}}{2}\right)^2$

This is a circle of radius  $\frac{\sqrt{3}}{2}$ , with center at  $(x, z) = (0, 0)$

(Use the approximation  $\frac{\sqrt{3}}{2} \approx 0.87$  when sketching the level curves for  $y = \pm 1$ )

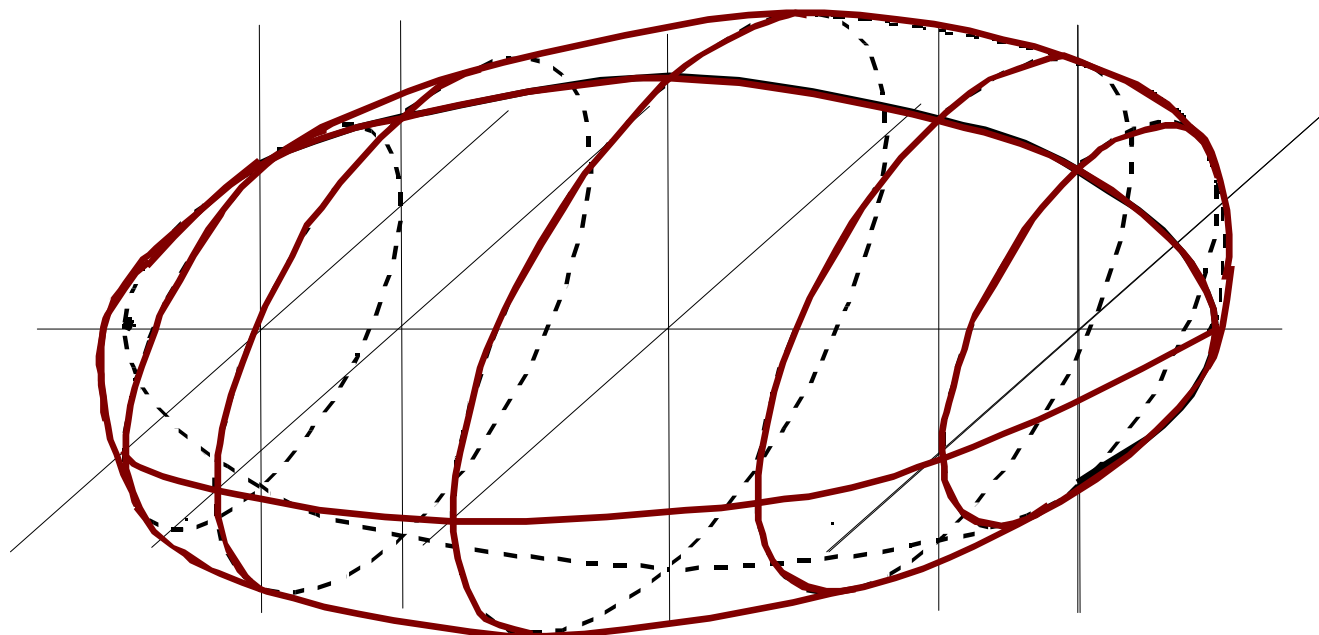
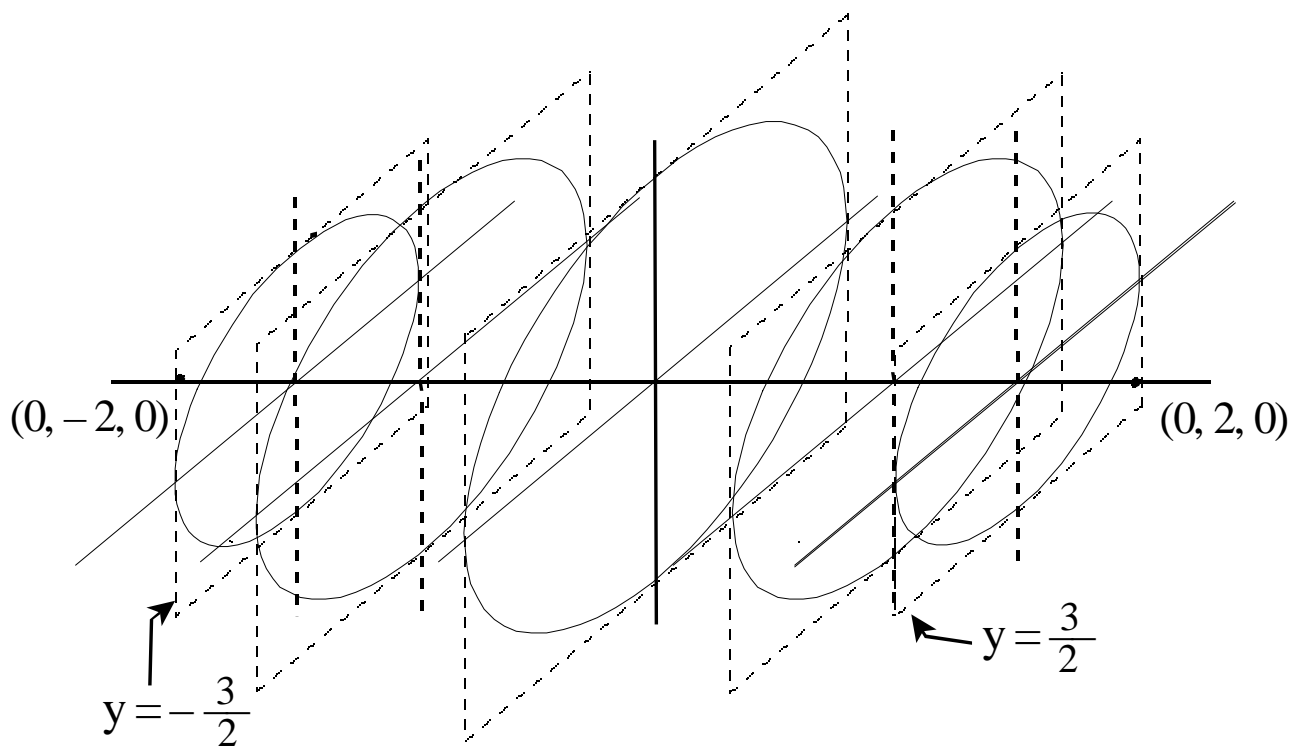


For  $y = \pm\frac{3}{2}$ ; the equation becomes  $x^2 + \left(\frac{9}{4}\right) + z^2 = 1 \Rightarrow x^2 + \left(\frac{9}{16}\right) + z^2 = 1 \Rightarrow x^2 + z^2 = \frac{7}{16}$

$$\Rightarrow x^2 + z^2 = \left(\frac{\sqrt{7}}{4}\right)^2$$

This is a circle of radius  $\frac{\sqrt{7}}{4}$ , with center at  $(x, z) = (0, 0)$

(Use the approximation  $\frac{\sqrt{7}}{4} \approx 0.67$  when sketching the level curves for  $y = \pm\frac{3}{2}$ )



15.  $16x^2 - y^2 + 16z^2 = 4$

$\Rightarrow 16x^2 + 16z^2 = y^2 + 4$

With the equation in this form, we can see that for a fixed value of  $y$ , the graph is a circle.

So, we will draw level curves for various values of  $y$

$y = 0$

$16x^2 + 16z^2 = 4 \Rightarrow x^2 + z^2 = \frac{1}{4} \Rightarrow x^2 + z^2 = \left(\frac{1}{2}\right)^2$

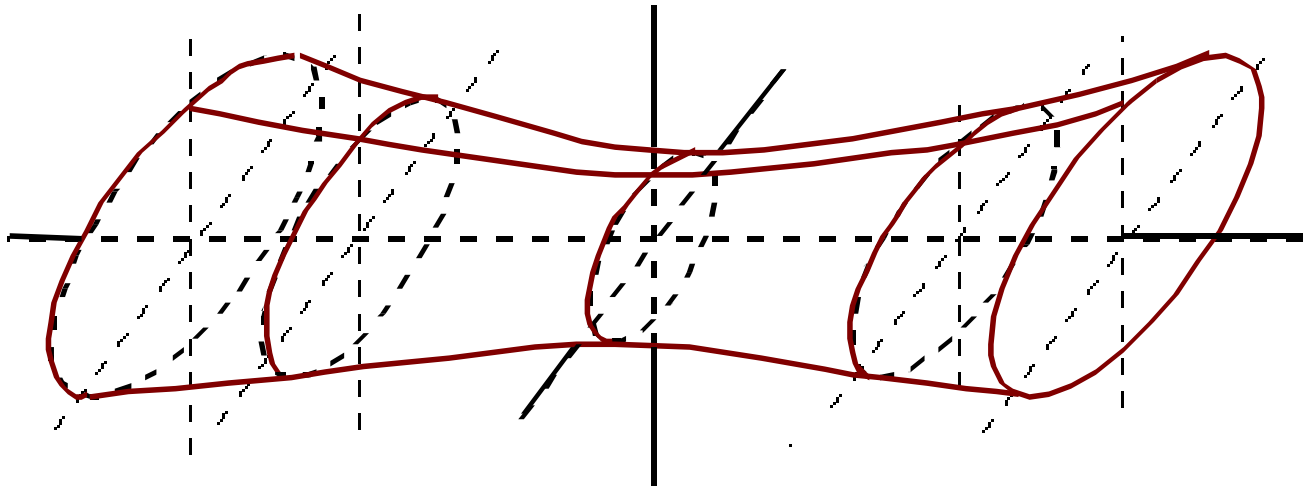
$y = \pm\sqrt{5} \quad (\sqrt{5} \approx 2.24)$

$16x^2 + 16z^2 = (\sqrt{5})^2 + 4 \Rightarrow 16x^2 + 16z^2 = 9 \Rightarrow x^2 + z^2 = \frac{9}{16} \Rightarrow x^2 + z^2 = \left(\frac{3}{4}\right)^2$

$y = \pm\sqrt{12} \quad (\sqrt{12} \approx 3.46)$

$16x^2 + 16z^2 = (\sqrt{12})^2 + 4 \Rightarrow 16x^2 + 16z^2 = 16 \Rightarrow x^2 + z^2 = 1$

Sketching the level curves and connecting the level curves lengthwise, we have:



17.  $4x^2 - y^2 - z^2 = 1$

19.  $x^2 - y + z^2 = 0$

21.  $x^2 - y^2 + z = 0$

23.  $z^2 = x^2 + \frac{y^2}{9}$