Polar Coordinates - Homework #1 Solutions

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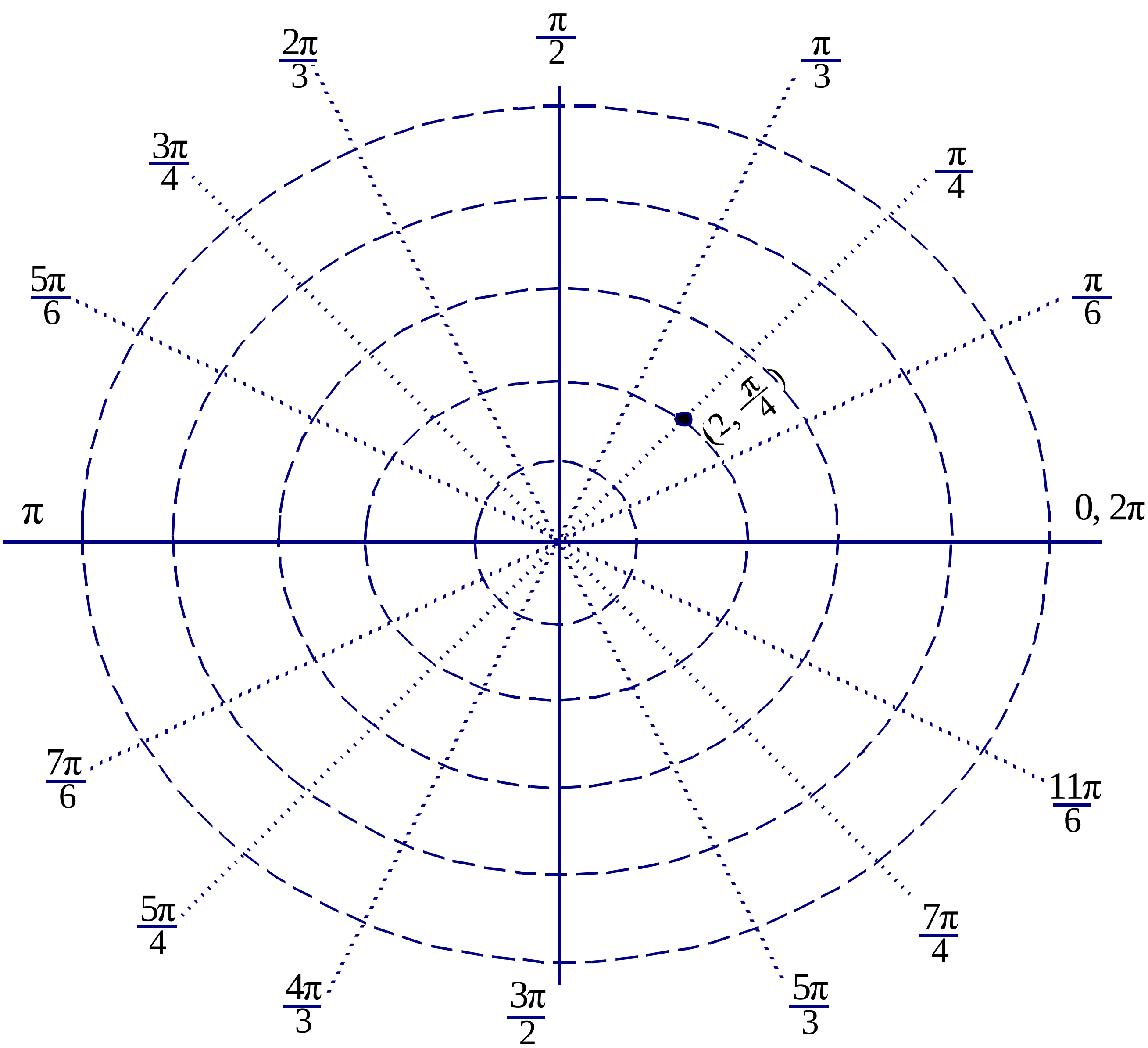
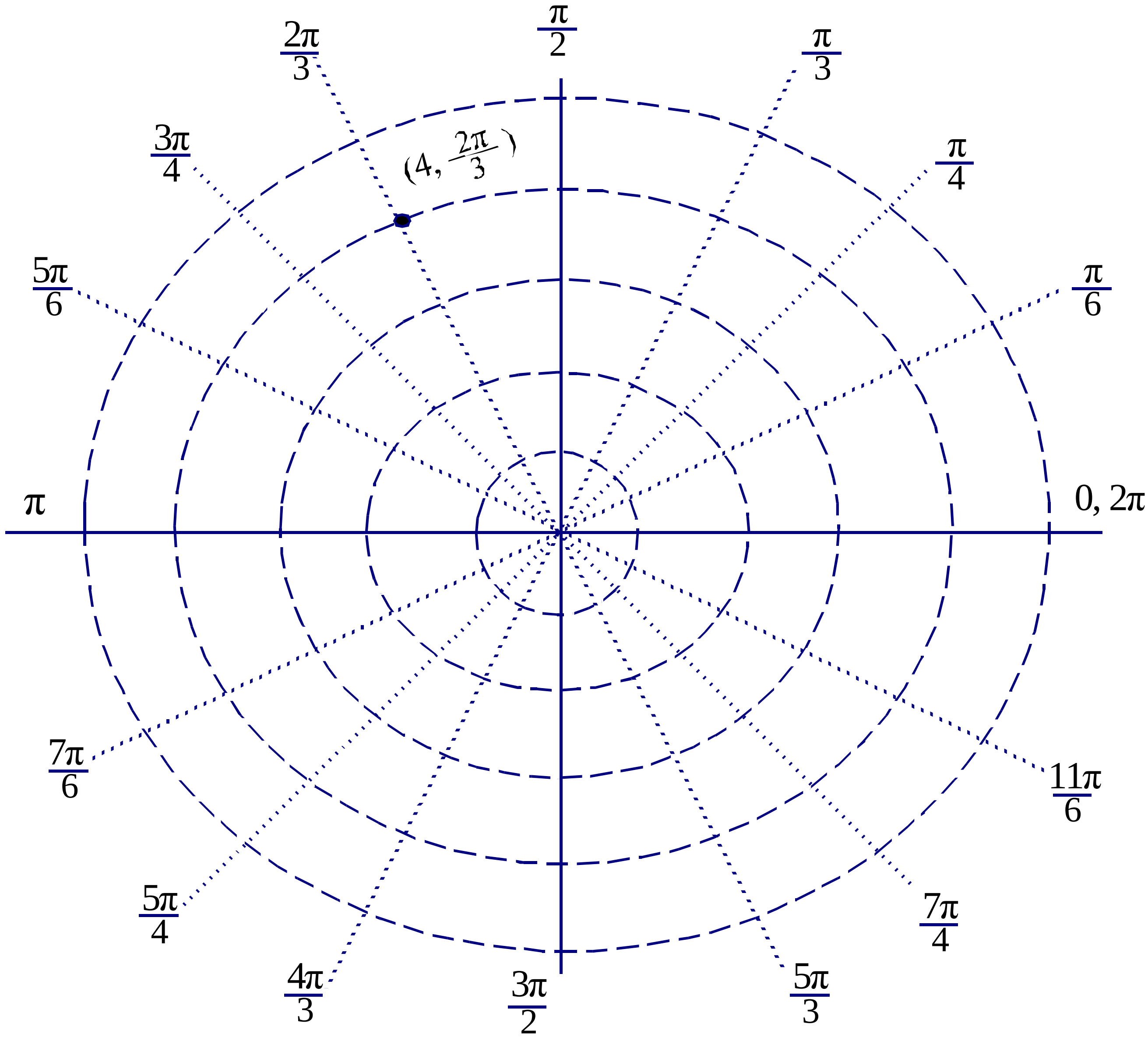
Pat Rossi Name ______________________

In Exercises 1-8, the points are expressed using polar coordinates. Graph the points. Feel free to use the polar grid template from my website (pat-rossi.com >> Academic Links >> MTH 2227 >> Handouts >> Polar Grid) to graph the points.

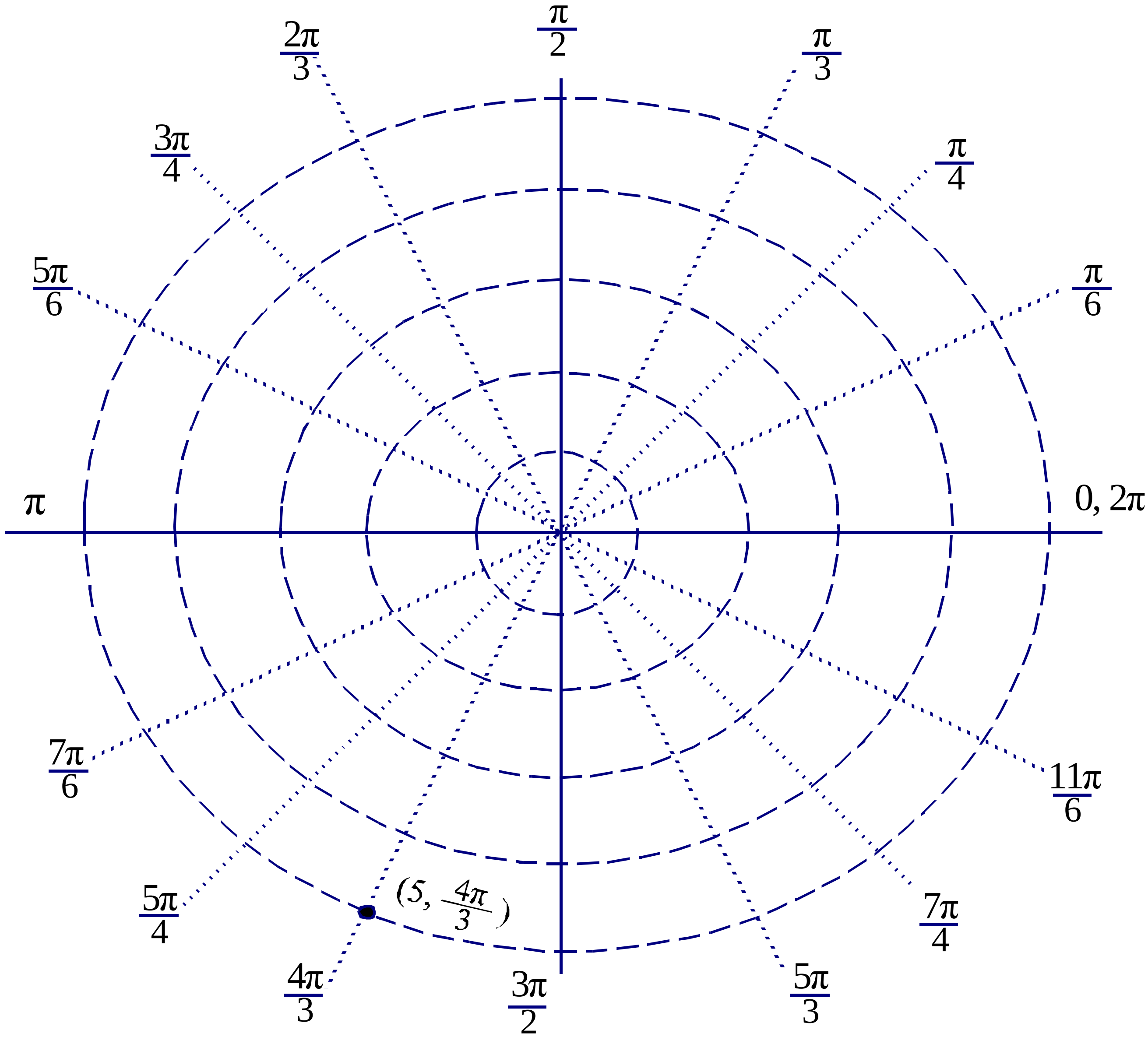
1. $(r, \theta) = \left(2, \frac{\pi}{4}\right)$



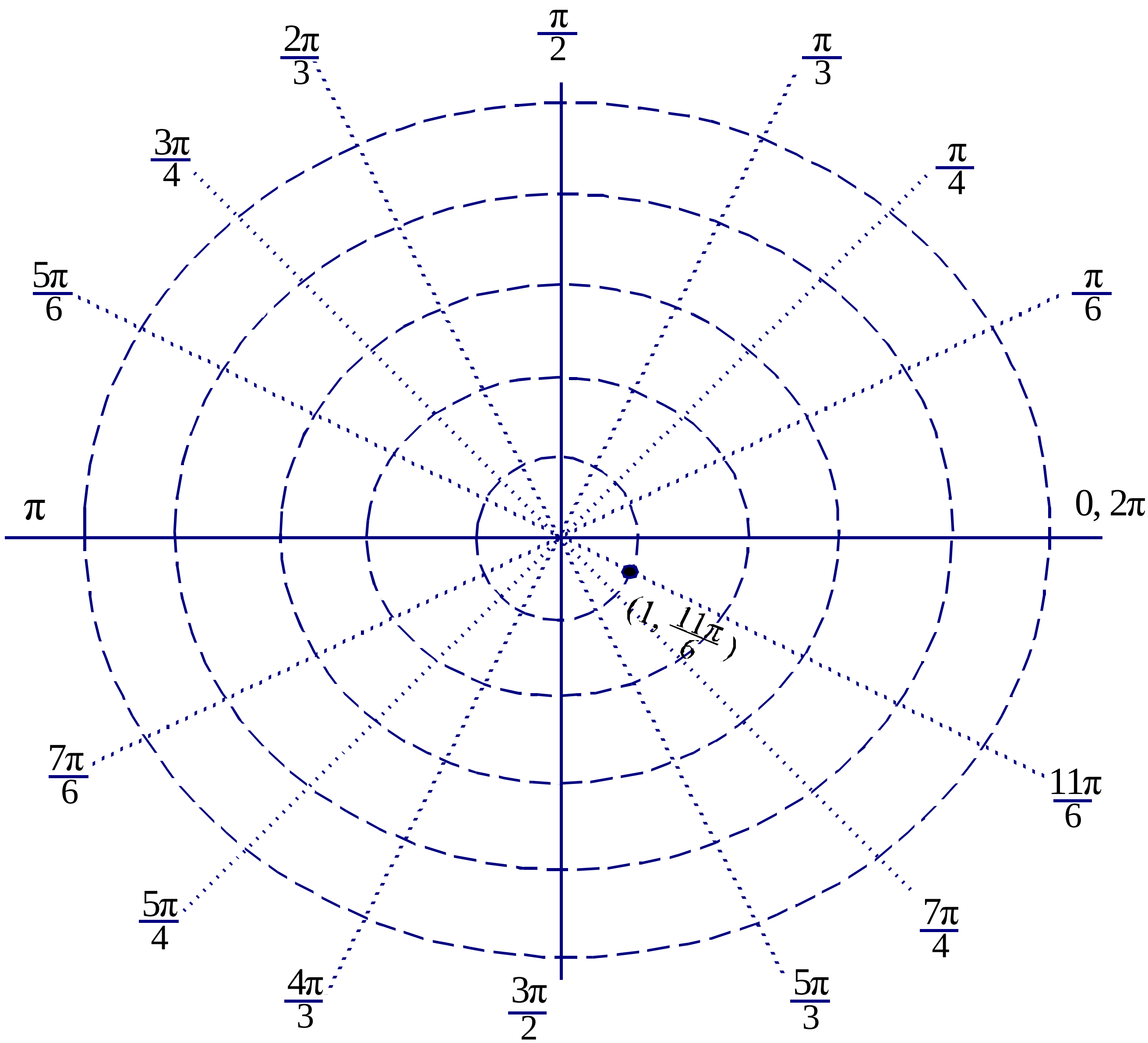
2. $(r, \theta) = \left(4, \frac{2\pi}{3}\right)$



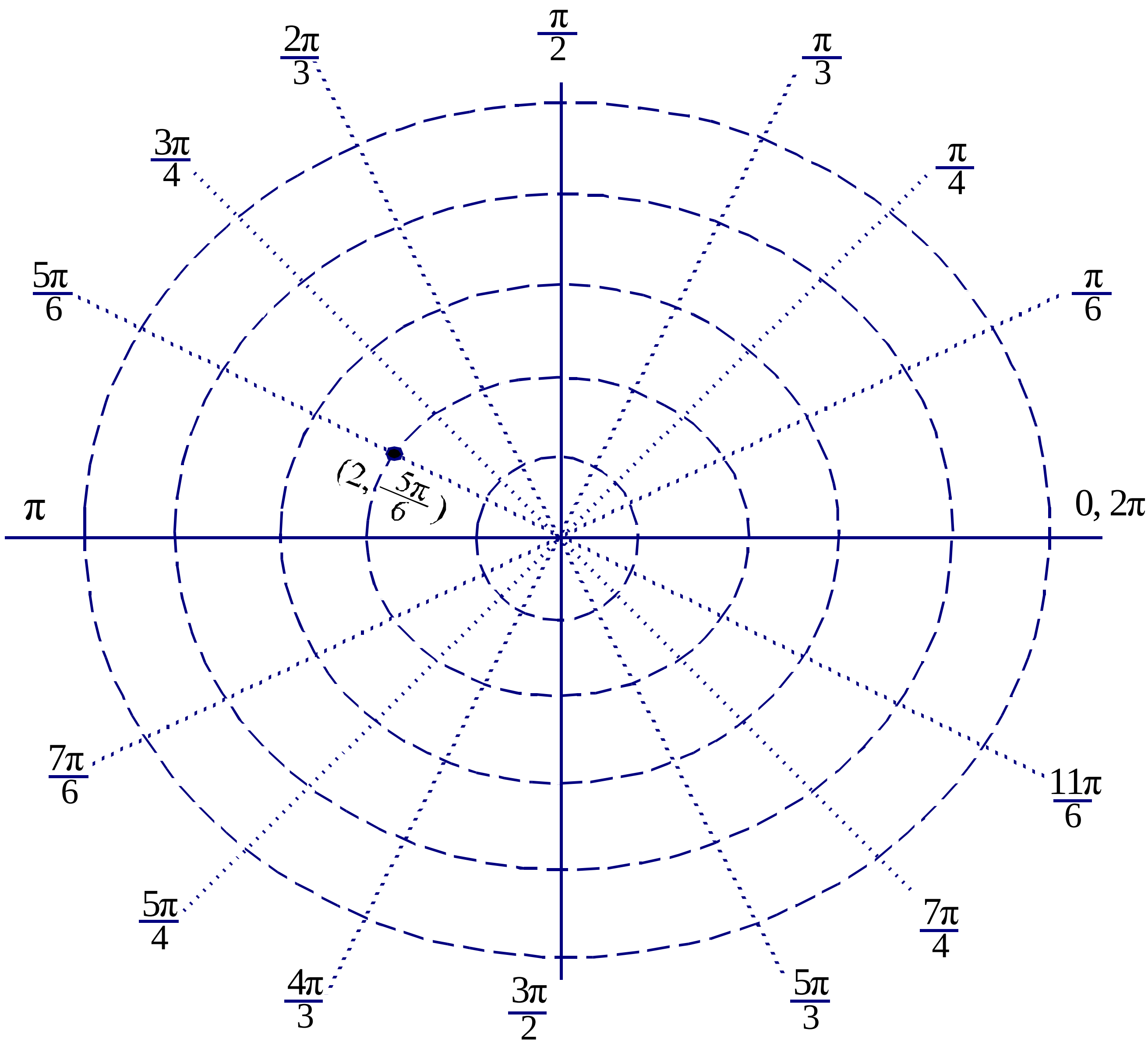
3. $(r, \theta) = \left(5, \frac{4\pi}{3}\right)$



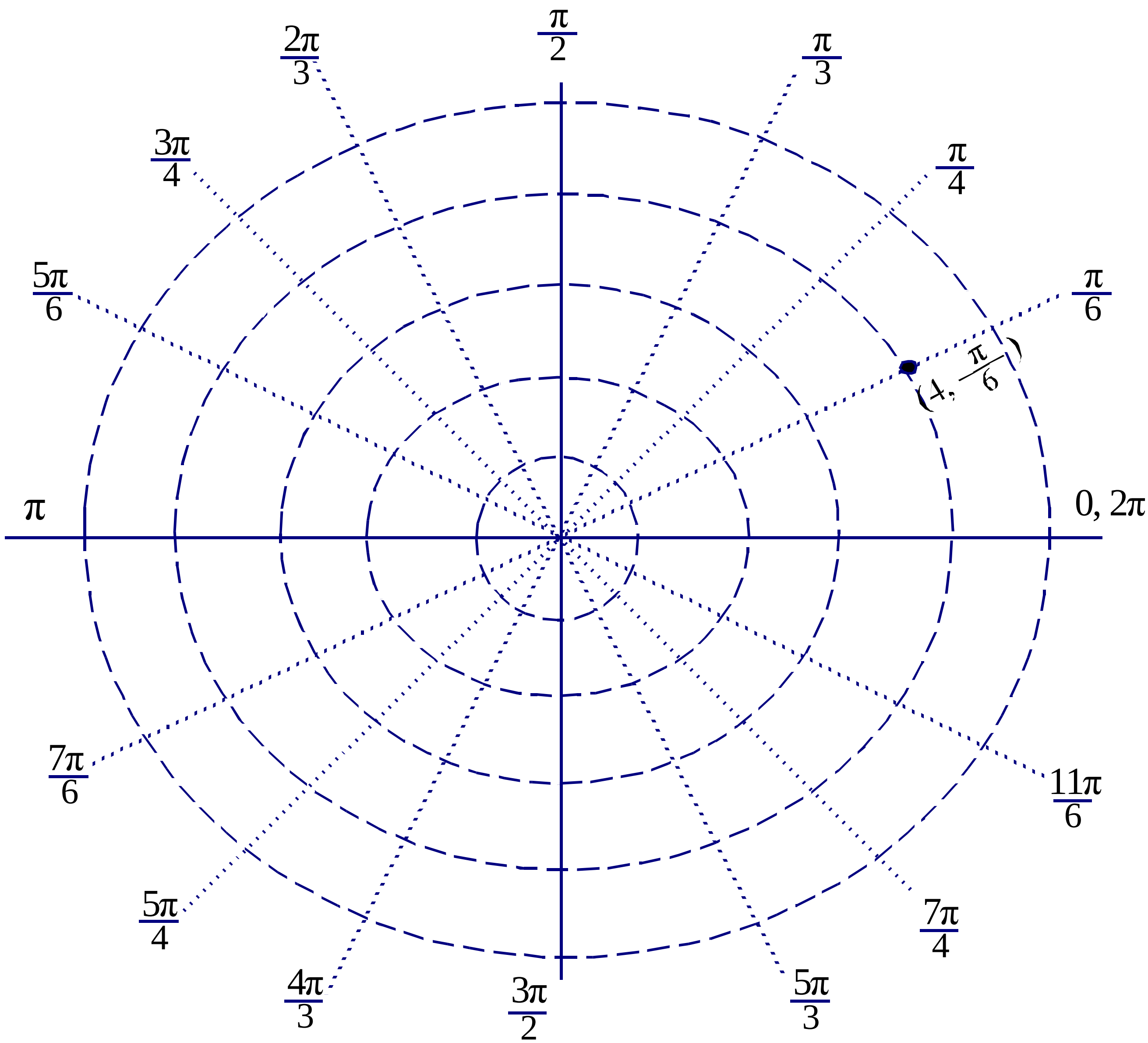
4. $(r, \theta) = \left(1, \frac{11\pi}{6}\right)$



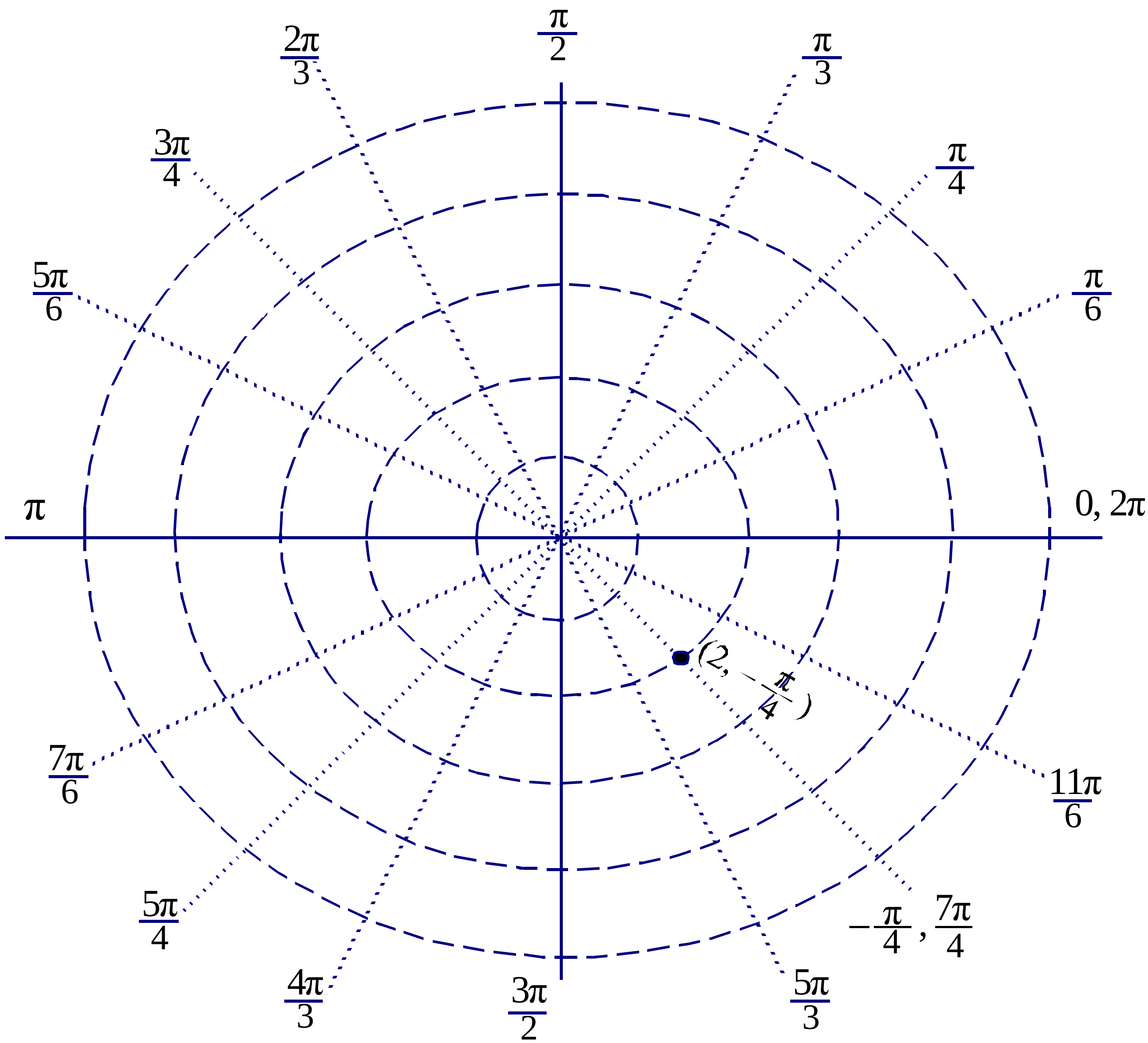
5. $(r,\theta) = \left(2, \frac{5\pi}{6}\right)$



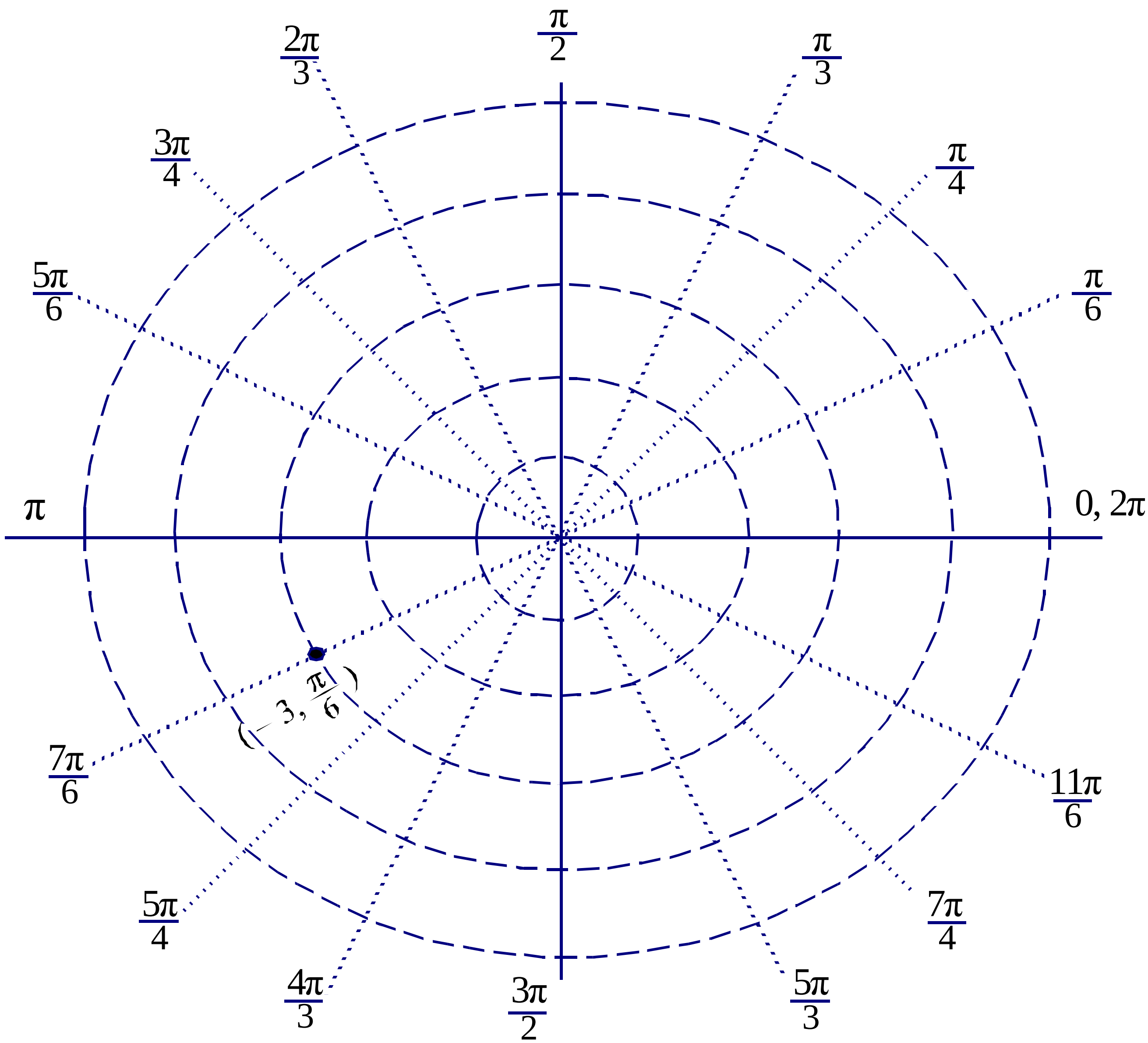
6. $(r, \theta) = \left(4, \frac{\pi}{6}\right)$



7. $(r, \theta) = \left(2, -\frac{\pi}{4}\right)$



8. $(r, \theta) = \left(-3, \frac{\pi}{6}\right)$



In Exercises 9-16, the given point is expressed in rectangular coordinates. Express the same point in polar coordinates.

9. $(x, y) = (4, 4)$

$r = \sqrt{x^2 + y^2} = \sqrt{(4)^2 + (4)^2} = \sqrt{32} = 4\sqrt{2}$

Since $(x, y) = (4, 4)$ is in the first quadrant, $\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{4}{4}\right) = \arctan(1) = \frac{\pi}{4}$

$$\boxed{(r, \theta) = \left(4\sqrt{2}, \frac{\pi}{4}\right)}$$

10. $(x, y) = \left(-\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$

$r = \sqrt{x^2 + y^2} = \sqrt{\left(-\frac{5}{2}\right)^2 + \left(\frac{5\sqrt{3}}{2}\right)^2} = \sqrt{\frac{25}{4} + \frac{75}{4}} = \sqrt{25} = 5$

Since $(x, y) = \left(-\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$ is in the second quadrant, $\theta = \arctan\left(\frac{y}{x}\right) + \pi = \arctan\left(\frac{\left(\frac{5\sqrt{3}}{2}\right)}{\left(-\frac{5}{2}\right)}\right) + \pi$

$= \arctan\left(-\sqrt{3}\right) + \pi = -\frac{\pi}{3} + \pi = \frac{2}{3}\pi$

$$\boxed{(r, \theta) = \left(5, \frac{2}{3}\pi\right)}$$

11. $(x, y) = (3, 0)$

$r = \sqrt{x^2 + y^2} = \sqrt{(3)^2 + (0)^2} = 3$

$\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{0}{3}\right) = \arctan(0) = 0$. This means that either $\theta = 0$, or $\theta = \pi$.

Since $(x, y) = (3, 0)$ is a point on the positive x-axis, $\theta = 0$.

$$\boxed{(r, \theta) = (3, 0)}$$

12. $(x, y) = (0, -4)$

$r = \sqrt{x^2 + y^2} = \sqrt{(0)^2 + (-4)^2} = 4$

$\theta = \arctan\left(\frac{4}{0}\right)$which is undefined. This means that either $\theta = \frac{\pi}{2}$ or $\theta = \frac{3\pi}{2}$.

Since $(x, y) = (0, -4)$ is on the negative y-axis, $\theta = -\frac{\pi}{2}$ or $\frac{3\pi}{2}$.

$$\boxed{(r, \theta) = \left(4, -\frac{\pi}{2}\right) \qquad (r, \theta) = \left(4, \frac{3\pi}{2}\right) \text{ is also acceptable}}$$

13. $(x,y) = \left(-\sqrt{3}, 1\right)$

$r = \sqrt{x^2+y^2} = \sqrt{\left(-\sqrt{3}\right)^2 + (1)^2} = \sqrt{3+1} = 2$

Since $(x,y) = \left(-\sqrt{3}, 1\right)$ is in the second quadrant, $\theta = \arctan\left(\frac{1}{\left(-\sqrt{3}\right)}\right) + \pi = \arctan\left(-\frac{1}{\sqrt{3}}\right) + \pi$

$= -\frac{\pi}{6} + \pi = \frac{5\pi}{6}$

$(r,\theta) = \left(2, \frac{5\pi}{6}\right)$

14. $(x,y) = (-2,-2)$

$r = \sqrt{x^2+y^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$

Since $(x,y) = (-2,-2)$ is in the third quadrant, $\theta = \arctan\left(\frac{y}{x}\right) + \pi = \arctan\left(\frac{-2}{-2}\right) + \pi = \arctan(1) + \pi$

$= \frac{\pi}{4} + \pi = \frac{5\pi}{4}$

$(r,\theta) = \left(2\sqrt{2}, \frac{5\pi}{4}\right)$

15. $(x,y) = \left(-6, -\frac{6}{\sqrt{3}}\right)$

$r = \sqrt{x^2+y^2} = \sqrt{(-6)^2 + \left(-\frac{6}{\sqrt{3}}\right)^2} = \sqrt{36 + \frac{36}{3}} = \sqrt{48} = 4\sqrt{3}$

Since $(x,y) = \left(-6, -\frac{6}{\sqrt{3}}\right)$ is in the third quadrant, $\theta = \arctan\left(\frac{y}{x}\right) + \pi = \arctan\left(\frac{\left(-\frac{6}{\sqrt{3}}\right)}{(-6)}\right) + \pi$

$= \arctan\left(\frac{1}{\sqrt{3}}\right) + \pi = \frac{\pi}{6} + \pi = \frac{7\pi}{6}$

$(r,\theta) = \left(4\sqrt{3}, \frac{7\pi}{6}\right)$

16. $(x,y) = \left(2, \frac{-2\sqrt{3}}{3}\right)$

$r = \sqrt{x^2+y^2} = \sqrt{(2)^2 + \left(\frac{-2\sqrt{3}}{3}\right)^2} = \sqrt{4 + \left(\frac{12}{9}\right)} = \sqrt{\frac{36}{9} + \left(\frac{12}{9}\right)} = \sqrt{\frac{48}{9}} = \frac{\sqrt{48}}{\sqrt{9}} = \frac{4\sqrt{3}}{3}$

Since $(x,y) = \left(2, \frac{-2\sqrt{3}}{3}\right)$ is in the fourth quadrant, $\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{\left(-2\sqrt{3}\right)}{2}\right)$

$= \arctan\left(-\sqrt{3}\right) = -\frac{\pi}{3}$

$(r,\theta) = \left(\frac{4\sqrt{3}}{3}, -\frac{\pi}{3}\right)$ $\quad$ $(r,\theta) = \left(\frac{4\sqrt{3}}{3}, \frac{5\pi}{3}\right)$ is also acceptable

In Exercises 17-24, the given point is expressed in polar coordinates. Express the same point in rectangular coordinates.

17. $(r, \theta) = \left(2, \frac{\pi}{4}\right)$

$x = r\cos(\theta) = 2\cos\left(\frac{\pi}{4}\right) = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$

$y = r\sin(\theta) = 2\sin\left(\frac{\pi}{4}\right) = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$

$$\boxed{(x, y) = \left(\sqrt{2}, \sqrt{2}\right)}$$

18. $(r, \theta) = \left(4, \frac{2\pi}{3}\right)$

$x = r\cos(\theta) = 4\cos\left(\frac{2\pi}{3}\right) = 4\left(-\frac{1}{2}\right) = -2$

$y = r\sin(\theta) = 4\sin\left(\frac{2\pi}{3}\right) = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

$$\boxed{(x, y) = \left(-2, 2\sqrt{3}\right)}$$

19. $(r, \theta) = \left(5, \frac{4\pi}{3}\right)$

$x = r\cos(\theta) = 5\cos\left(\frac{4\pi}{3}\right) = 5\left(-\frac{1}{2}\right) = -\frac{5}{2}$

$y = r\sin(\theta) = 5\sin\left(\frac{4\pi}{3}\right) = 5\left(-\frac{\sqrt{3}}{2}\right) = -\frac{5\sqrt{3}}{2}$

$$\boxed{(x, y) = \left(-\frac{5}{2}, -\frac{5\sqrt{3}}{2}\right)}$$

20. $(r, \theta) = \left(1, \frac{11\pi}{6}\right)$

$x = r\cos(\theta) = 1 \cdot \cos\left(\frac{11\pi}{6}\right) = 1 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

$y = r\sin(\theta) = 1 \cdot \sin\left(\frac{11\pi}{6}\right) = 1 \cdot \left(-\frac{1}{2}\right) = -\frac{1}{2}$

$$\boxed{(x, y) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)}$$

21. $(r, \theta) = \left(2, \frac{5\pi}{6}\right)$

$x = r\cos(\theta) = 2\cos\left(\frac{5\pi}{6}\right) = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

$y = r\sin(\theta) = 2\sin\left(\frac{5\pi}{6}\right) = 2\left(\frac{1}{2}\right) = 1$

$$\boxed{(x, y) = \left(-\sqrt{3}, 1\right)}$$

22. $(r, \theta) = \left(4, \frac{\pi}{6}\right)$

$x = r\cos(\theta) = 4\cos\left(\frac{\pi}{6}\right) = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

$y = r\sin(\theta) = 4\sin\left(\frac{\pi}{6}\right) = 4\left(\frac{1}{2}\right) = 2$

$$\boxed{(x, y) = \left(2\sqrt{3}, 2\right)}$$

23. $(r, \theta) = \left(2, -\frac{\pi}{4}\right)$

$x = r\cos(\theta) = 2\cos\left(-\frac{\pi}{4}\right) = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$

$y = r\sin(\theta) = 2\sin\left(-\frac{\pi}{4}\right) = 2\left(-\frac{\sqrt{2}}{2}\right) = -\sqrt{2}$

$$\boxed{(x, y) = \left(\sqrt{2}, -\sqrt{2}\right)}$$

24. $(r, \theta) = \left(-3, \frac{\pi}{6}\right)$

$x = r\cos(\theta) = -3\cos\left(\frac{\pi}{6}\right) = -3\left(\frac{\sqrt{3}}{2}\right) = -\frac{3\sqrt{3}}{2}$

$y = r\sin(\theta) = -3\sin\left(\frac{\pi}{6}\right) = -3\left(\frac{1}{2}\right) = -\frac{3}{2}$

$$\boxed{(x, y) = \left(-\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)}$$