MTH 263 Practice Test #1

SPRING 1999

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Instructions This may take longer than an hour to complete. The ANSWER section follows. The SOLUTION section follows the ANSWER section.

1. Find the area of the region bounded by the graph $r = 2a\cos(\theta)$.

2. Find the area of the region that is inside the "cardioid" $r = a(1 + \cos(\theta))$, but outside the circle $r = a$.

3. Find the area bounded by the graph of $r^2 = 4\sin(2\theta)$.

4. Compute the norm of $\vec{\mathbf{v}} = (1, 3, 5)$

5. Find a unit vector in $\Re^3$ having the same direction as $\vec{\mathbf{v}} = \langle 1, 3, 5\rangle$.

6. Find a unit vector in $\Re^2$ having the same direction as $\vec{\mathbf{v}} = \langle 1, 2\rangle$

7. Given vectors $\vec{\mathbf{u}} = \langle 1, 2\rangle$ and $\vec{\mathbf{v}} = \langle 4, 3\rangle$, compute the angle θ between $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$.

8. Given vectors $\vec{\mathbf{u}} = \langle 1, 2\rangle$ and $\vec{\mathbf{v}} = \langle 4, 3\rangle$, compute the projection of $\vec{\mathbf{u}}$ onto $\vec{\mathbf{v}}$, and the projection of $\vec{\mathbf{u}}$ orthogonal to $\vec{\mathbf{v}}$, $\mathbf{orth}_{\vec{\mathbf{v}}}\vec{\mathbf{u}}$.

9. Show that the vector $\vec{\mathbf{v}}$ with initial point $(4, 2)$ and terminal point $(8, 6)$ is parallel to the vector $\vec{\mathbf{u}}$ whose initial point is $(0, 0)$ and whose terminal point $(4, 4)$.

10. Show that the vectors $\vec{\mathbf{u}} = \langle 3, 2\rangle$ and $\vec{\mathbf{v}} = \langle 2, -3\rangle$ are perpendicular.

11. Let $\vec{\mathbf{v}} = 3\vec{i} - 6\vec{j} + 4\vec{k}$ and $\vec{\mathbf{u}} = \vec{i} + 4\vec{j} + 4\vec{k}$.

    (a) Compute $||\vec{\mathbf{v}}||$

    (b) Compute $||3\vec{\mathbf{v}} - \vec{\mathbf{u}}||$

12. Find the angle between the line containing the points $(1, 2, 3)$ and $(2, 4, 6)$, and the line containing the points $(1, 2, 3)$ and $(3, 6, 9)$.

13. Given that $\vec{\mathbf{u}} = \langle 3, -1, 2\rangle$ and $\vec{\mathbf{v}} = \langle 4, 1, -5\rangle$, compute:

(a) $comp_u(\vec{\mathbf{v}})$

(b) $proj_u(\vec{\mathbf{v}})$.

(c) and $orth_u(\vec{\mathbf{v}})$.

14. $\vec{\mathbf{a}} = 7\vec{i} + 6\vec{j} + 5\vec{k}$ and $\vec{\mathbf{b}} = -\vec{i} + 2\vec{j} - 3\vec{k}$. Compute $\vec{\mathbf{a}} \times \vec{\mathbf{b}}$.

15. Find the volume of the parallelepiped with vertices $(1, -3, 2)$, $(8, 3, 7)$, $(0, -1, -1)$, and $(4, 2, 1)$. (Hint: You may be able to use the results of problem #1.)

16. Compute the area of the parallelogram with vertices $(1, 2)$, $(2, 6)$, and $(6, 5)$.