MTH 263 Practice Test #2 - Solutions

SPRING 1999

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1. Find the parametric equations of the line containing the points $(3,-7,4)$ and $(2,1,2)$.

   Given the line containing the points $(3,-7,4)$ and $(2,1,2)$, the vector $\vec{\mathbf{v}} = \langle (2-3), (1-(-7)), (2-4) \rangle = \langle -1, 8, -2 \rangle$ is parallel to the line.

   To get the parametric equations of the line, we use the form

$$x = x_0 + at$$
$$y = y_0 + bt$$
$$z = z_0 + ct$$

   where (x_0, y_0, z_0) is a point on the line, and $\vec{\mathbf{v}} = \langle a, b, c \rangle$ is a vector parallel to the line.

   Letting (x_0, y_0, z_0) be the point $(3,-7,4)$, we have:

$$x = 3 - t$$
$$y = -7 + 8t$$
$$z = 4 - 2t$$

2. Find an equation of the plane containing the point $(4,2,1)$ and having the normal vector $\vec{\mathbf{n}} = \langle 5, 5, 3 \rangle$.

   Here, we can use the form:

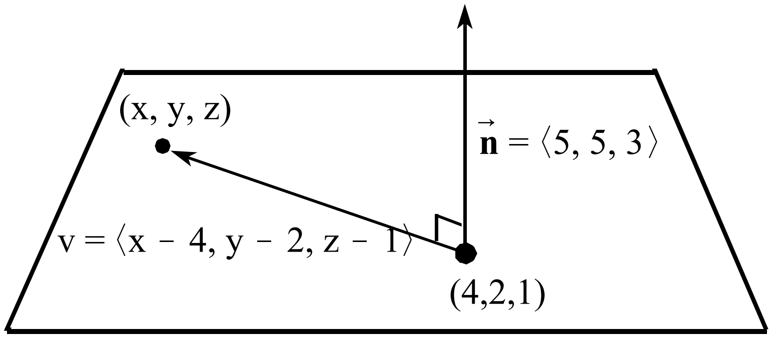
$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

   where (x_0, y_0, z_0) is a point in the plane, and $\vec{\mathbf{n}} = \langle a, b, c \rangle$ is a vector normal to the plane. Letting (x_0, y_0, z_0) be the point $(4,2,1)$, we have:

$$5(x-4) + 5(y-2) + 3(z-1) = 0$$

$$\text{or } 5x + 5y + 3z = 33$$

   Alternately, if (x, y, z) is any point in the plane, then the vector $\vec{\mathbf{v}} = \langle x-4, y-2, z-1 \rangle$ is in the plane (see below).



Since $\vec{\mathbf{n}} = \langle 5, 5, 3 \rangle$ is normal to the plane,it is also normal to $\vec{\mathbf{v}} = \langle x-4, y-2, z-1 \rangle$. This implies that $\vec{\mathbf{n}} \circ \vec{\mathbf{v}}$ =0.

$\Rightarrow \langle 5, 5, 3 \rangle \circ \langle x-4, y-2, z-1 \rangle = 5(x-4) + 5(y-2) + 3(z-1) = 0$

$\Rightarrow 5x + 5y + 3z = 33$

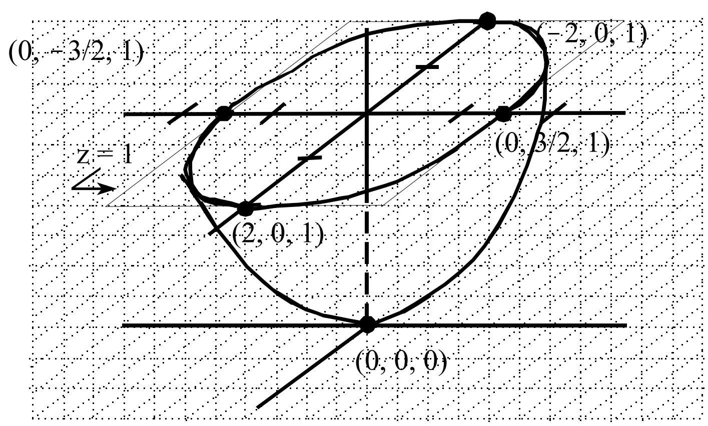
3. Graph the surface given by the equation $9x^2 + 16y^2 - 36z = 0$.

   First, a few observations:

   1. When $z < 0$, we have $9x^2 + 16y^2 < 0$, which is an impossibility. Thus, the surface must lie on, or above, the $x - y$ plane.
   2. When $z = 0$, we have $9x^2 + 16y^2 = 0$, so $x = 0$, and $y = 0$. (i.e., the graph comes to a point at the origin.
   3. When $z = 1$, we have $9x^2 + 16y^2 = 36 \Rightarrow \frac{x^2}{(2)^2} + \frac{y^2}{\left(\frac{3}{2}\right)^2} = 1$

      This is an ellipse in the plane $z = 1$, with center $(0, 0, 1)$, vertices on the "x-axis" $(\pm 2, 0, 1)$, and vertices on the "y-axis" $\left(0, \pm\frac{3}{2}, 1\right)$.

   The graph is drawn below:



4. Compute:

$$\int_0^1 \int_{-1}^1 \int_1^2 \left(x^2 + yz\right) dz\, dx\, dy$$

$\int_{y=0}^{y=1} \int_{x=-1}^{x=1} \int_{z=1}^{z=2} (x^2 + yz)\, dz\, dx\, dy = \int_{y=0}^{y=1} \int_{x=-1}^{x=1} \left[x^2 z + \frac{1}{2} y z^2\right]_{z=1}^{z=2} dx\, dy =$

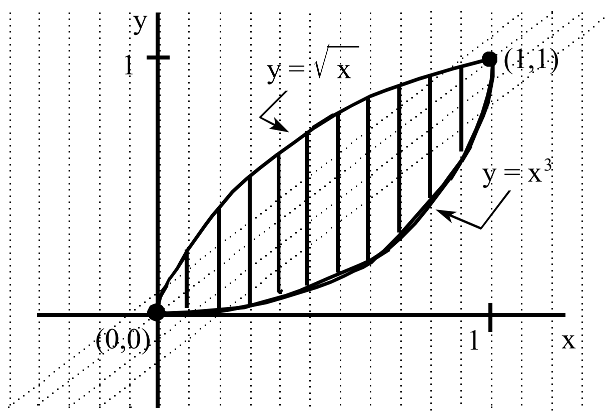
$\int_{y=0}^{y=1} \int_{x=-1}^{x=1} \left[\left(x^2 (2) + \frac{1}{2} y (2)^2\right) - \left(x^2 (1) + \frac{1}{2} y (1)^2\right)\right] dx\, dy = \int_{y=0}^{y=1} \int_{x=-1}^{x=1} \left(x^2 + \frac{3}{2} y\right) dx\, dy =$

$\int_{y=0}^{y=1} \left[\frac{1}{3} x^3 + \frac{3}{2} y x\right]_{x=-1}^{x=1} dy = \int_{y=0}^{y=1} \left[\left(\frac{1}{3} (1)^3 + \frac{3}{2} y (1)\right) - \left(\frac{1}{3} (-1)^3 + \frac{3}{2} y (-1)\right)\right] dy =$

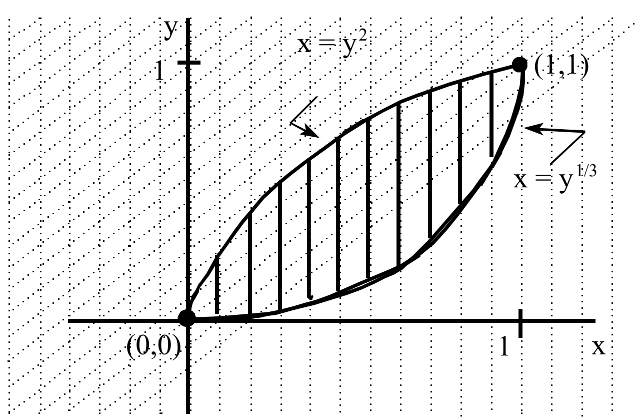
$\int_{y=0}^{y=1} \left(\frac{2}{3} + 3y\right) dy = \left[\frac{2}{3} y + \frac{3}{2} y^2\right]_0^1 = \left(\frac{2}{3} (1) + \frac{3}{2} (1)^2\right) - \left(\frac{2}{3} (0) + \frac{3}{2} (0)^2\right) = \frac{13}{6}$

5. Given $\int_0^1 \int_{x^3}^{\sqrt{x}} y\, dy\, dx$, **reverse** the order of integration, and then integrate.

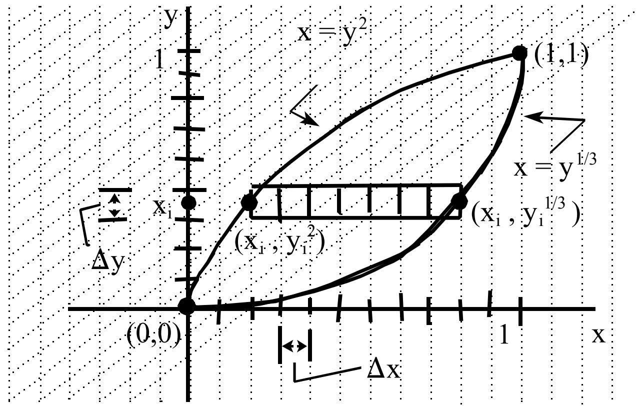
   1. First, we sketch the region of integration.



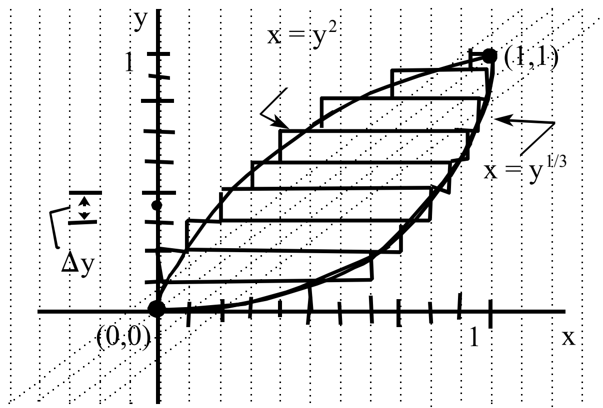
   Rewrite the equations defining the boundaries of the region in terms of the "other" variable.



   2. Determine the limits on the integrals. Note that if we integrate with respect to x first, then we are "adding up the volumes of columns" from $x = y^2$ to $x = y^{\frac{1}{3}}$.



3. Finally, note that as we add up the volume of slices in the y direction, we "add up the volumes of slices" from $y = 0$ to $y = 1$.



Thus our integral is

$$\int_{y=0}^{y=1} \int_{x=y^2}^{x=y^{\frac{1}{3}}} y\, dx\, dy$$

Computing, we have: $\int_{y=0}^{y=1} \int_{x=y^2}^{x=y^{\frac{1}{3}}} y\, dx\, dy = \int_{y=0}^{y=1} [yx]_{x=y^2}^{=y^{\frac{1}{3}}}\, dy =$
$\int_{y=0}^{y=1} \left[\left(y\left(y^{\frac{1}{3}}\right)\right) - \left(y\left(y^2\right)\right)\right] dy = \int_{y=0}^{y=1} \left(y^{\frac{4}{3}} - y^3\right) dy =$
$\left[\frac{3}{7}y^{\frac{7}{3}} - \frac{1}{4}y^4\right]_0^1 = \left(\frac{3}{7}(1)^{\frac{7}{3}} - \frac{1}{4}(1)^4\right) - \left(\frac{3}{7}(0)^{\frac{7}{3}} - \frac{1}{4}(0)^4\right) = \frac{5}{28}$