MTH 3331 Practice Test #1

SUMMER 2013

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Instructions. Answers follow this section. Solutions follow the Answers section.

1. Given

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 0 & 4 \end{bmatrix} \qquad D = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

   Compute the following where possible:

   (a) $A + B$
   (b) $A^T + B^T$
   (c) AC
   (d) CA
   (e) CD
   (f) BC
   (g) $(A + B)\,C$
   (h) $A + D$
   (i) AD

2. For $(n \times n)$ matrices A, B, C, write the transpose of the following:

   (a) A^T
   (b) $A\,(B + C)$

3. Write the system of equations

$$\begin{array}{rrrcl} 4x & -7y & +3z & = & 9 \\ -2x & & +z & = & 5 \\ 8x & +3y & +2z & = & 2 \end{array}$$

   in the form: $\begin{bmatrix} & A & \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 2 \end{bmatrix}$

4. For any square $(n \times n)$ matrix A, show that $A^3 - I = (A - I)\,(A^2 + A + I)$ (Hint: Don't try to do this using individual matrix entries a_{ij}. Instead, use A and I, and the algebraic properties of matrices.

5. For $(n \times n)$ matrices A and B, compute $(A - B)\,(A^2 + AB + B^2)$.

6. If matrices A, B, C are such that A is nonzero and $AB = AC$, is it necessarily true that $B = C$? Why or why not?

7. Define Upper Triangular Matrix.

8. Define Lower Triangular Matrix.

9. Define Diagonal Matrix.

10. What is the multiplicative identity for $n \times n$ matrices?

11. For
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \qquad C = \begin{bmatrix} 7 & 6 & 3 \\ 2 & 1 & 0 \\ 9 & 8 & 5 \end{bmatrix}$$
compute $4A + 2B - C$.

12. Calculate A^{-1} if
$$A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

13. Given
$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{bmatrix} \qquad E = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$
   (a) Compute AB. What does this tell us about B?
   (b) Compute $A + E$
   (c) Compute $BA + BE$

14. Given
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix}$$
   (a) Compute AB
   (b) What does the result in problem 14.a tell us about matrix multiplication?

15. Simplify (i.e. State in terms of A, B, and C only):
$\left[\left(A^T + B^T\right) \cdot C^T\right]^T =$