MTH 3331 Practice Test #2

SUMMER 2013

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1. Given the vectors $\tilde{\mathbf{u}} = (a, b)$ and $\tilde{\mathbf{v}} = (c, d)$, give the geometric interpretation of the following:

   (a) $\tilde{\mathbf{u}} + \tilde{\mathbf{v}}$

   (b) $\tilde{\mathbf{u}} - \tilde{\mathbf{v}}$

2. Find the norm or "length" of the vectors $\tilde{\mathbf{u}} = (3, 4, 12)$ and $\tilde{\mathbf{v}} = (4, 4, 5, -2)$

3. State the Cauchy-Schwarz Inequality.

4. State the Triangle Inequality and give its geometric interpretation.

5. Given that $\tilde{\mathbf{u}} = (1, 2, 3, 4)$ and $\tilde{\mathbf{v}} = (3, 1, 2, 1)$, compute the dot product $\mathbf{u} \circ \mathbf{v}$.

6. Given that $\tilde{\mathbf{u}} = \left(\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}}\right)$,

   (a) compute $||\tilde{\mathbf{u}}||$

   (b) What does this tell us about $\tilde{\mathbf{u}}$?

7. Find $\cos(\theta)$, where θ is the angle between vectors $\tilde{\mathbf{u}} = (2, 4, 1)$ and $\tilde{\mathbf{v}} = (-2, 2, -4)$

8. Find $\cos(\theta)$, where θ is the angle between vectors $\tilde{\mathbf{u}} = (2, 1, 4, 7)$ and $\tilde{\mathbf{v}} = (1, -1, 2, -1)$

9. Given the system of equations $\begin{bmatrix} 1 & 4 & 2 \\ 2 & 5 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}$, the row reduced form of the corresponding augmented matrix is $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$. Give the general solution of the system.

10. Assume that each of the following matrices is the matrix of coefficients of a homogeneous system of equations. Decide whether each one has any solution other than the trivial solution. If so, give the solution.

    (a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

    (b) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

    (c) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

11. If the system of equations $A\tilde{\mathbf{x}} = \tilde{\mathbf{b}}$ has a unique solution, what is the solution set of $A\tilde{\mathbf{x}} = \mathbf{0}$?

12. Find the general solution of the system: $\begin{array}{rcrcrcr} 2x & + & 3y & + & 4z & = & 2 \\ 4x & + & 5y & + & z & = & 3 \end{array}$

13. Consider the system of equations $A\tilde{\mathbf{x}} = \tilde{\mathbf{b}}$. Show that if $\tilde{\mathbf{x}}_1$ and $\tilde{\mathbf{x}}_2$ are solutions of the system $A\tilde{\mathbf{x}} = \tilde{\mathbf{b}}$, then $(\tilde{\mathbf{x}}_1 - \tilde{\mathbf{x}}_2)$ is a solution of the system $A\tilde{\mathbf{x}} = \tilde{\mathbf{0}}$.

14. If $\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \ldots, \tilde{\mathbf{x}}_k$ are solutions of the system $A\tilde{\mathbf{x}} = \tilde{\mathbf{0}}$, show that every linear combination of $\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \ldots, \tilde{\mathbf{x}}_k$ is also a solution of the system $A\tilde{\mathbf{x}} = \tilde{\mathbf{0}}$.

15. The augmented matrix from a system of equations is given in reduced form below. Write the solution of the system as the sum of a particular solution plus the homogeneous solution.

$$\left[\begin{array}{cccccr|r} 1 & 2 & 0 & 1 & 0 & -1 & 6 \\ 0 & 0 & 1 & 4 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

16. If $\tilde{\mathbf{x}}_1$ is a solution of the system of equations $A\tilde{\mathbf{x}} = \tilde{\mathbf{b}}$, and $\tilde{\mathbf{y}}_1$ is a solution of the system of equations $A\tilde{\mathbf{y}} = \tilde{\mathbf{0}}$, for some matrix A, what can be said about $\tilde{\mathbf{x}}_1 + \tilde{\mathbf{y}}_1$?

17. Write the parametric equations of the line through the point $(1, 4)$ and parallel to the vector $\tilde{\mathbf{v}} = (2, 2)$.

18. Let $p_1 = (2, 2, 2)$ and $p_2 = (3, 5, 6)$. Write the equation of the plane containing the point p_1 and normal to the vector $p_1 p_2$ (i.e normal to the vector with endpoints p_1 and p_2.)

19. Write the equation of the plane in $\mathbf{R}^3$ which contains the point $(1, 1, 1)$, and is parallel to the plane given by the equation $2x + 2y - 3z = 16$.

20. Write the parametric equations for the line which contains the point $(2, 1, 2)$ and which is parallel to the vector $(-1, 3, 3)$.

21. Compute A^{-1} if $A = \begin{bmatrix} 1 & 4 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

22. Use the result of problem #21 to solve the system:

$$\begin{array}{rcrcrcr} x & + & 4y & - & z & = & 4 \\ x & + & y & + & z & = & 2 \\ 2x & & & + & 3z & = & -1 \end{array}$$

23. Let $p_1 = (1, 1, 1)$ and $p_2 = (3, 4, 5)$. Write the equation of the plane which contains the point p_1 and is normal to the vector $p_1 p_2$ (i.e normal to the vector with endpoints p_1 and p_2.)

24. Write the equation of the plane in $\mathbf{R}^3$ which contains the point $(2, 1, 2)$, and is parallel to the plane given by the equation $4x - 4y + 3z = 20$.

25. In each case below, the augmented matrix of a system of equations is given in row reduced form. Write the solution of the system as the sum of $n \times 1$ arrays. Do this by finding a particular solution and finding the homogeneous solution, and then adding the two together.

(a) $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(b) $\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$

(c) $\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$