MTH #3331 Practice Test #3 - Answers

Summer 2013

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1. Given vectors $\mathbf{u} = (2,3)$ and $\mathbf{v} = (1,1)$, compute:

   (a) $\mathbf{proj_v}(\mathbf{u}) = \left(\frac{5}{2}, \frac{5}{2}\right)$.

   (b) $\mathbf{orth_v}(\mathbf{u}) = \left(-\frac{1}{2}, \frac{1}{2}\right)$.

2. ~

   (a) -52

   (b) -156

3. ~

   (a) 0

   (b) 0

   (c) 0

   (d) 0

   (e) 0

   (f) 0

4. No. Consider, for example, $A = \begin{bmatrix} 2 & 4 \\ 4 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$.

5. ~

   (a) $\det\left(\begin{bmatrix} 1-k & 2+0 & 3+0 \\ 4 & 1 & 7 \\ 2 & 8 & 9 \end{bmatrix}\right) = \det\left(\begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 7 \\ 2 & 8 & 9 \end{bmatrix}\right) + \det\begin{pmatrix} -k & 0 & 0 \\ 4 & 1 & 7 \\ 2 & 8 & 9 \end{pmatrix}$

   $= -1 + 47k$

   (b) $\det\left(\begin{bmatrix} 1-k & 4-2 & 1+2 \\ 4 & 1 & 7 \\ 2 & 8 & 9 \end{bmatrix}\right) = \det\left(\begin{bmatrix} 1 & 4 & 1 \\ 4 & 1 & 7 \\ 2 & 8 & 9 \end{bmatrix}\right) + \det\begin{pmatrix} -k & -2 & 2 \\ 4 & 1 & 7 \\ 2 & 8 & 9 \end{pmatrix}$

   $= -1 + 47k$

6. 3
7. ~
   (a) 0
   (b) 3
   (c) 1
8. -24
9. $\det(A) = 12$; $\det(B) = 6$; $\det(AB) = 72$
10. ~
    (a) Singular.
    (b) Singular.
    (c) Non-singular.
    (d) Non-singular.
11. ~
    (a) Non-singular.
    (b) Singular.
    (c) Non-singular.
    (d) Singular
12. $\text{Rank}(A) = 6$.
13. $\text{Rank}(A) = 5$.
14. See Solutions.
15. See Solutions.
16. See Solutions.

17. For $\lambda = 1, 3$. For the eigenvalue $\lambda_1 = 1$, we have the eigenvector $k \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

For the eigenvalue $\lambda_2 = 3$,we have the eigenvector $k \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

18. ~

(a) $\lambda = 2, 7$ are eigenvalues.

For the eigenvalue $\lambda_1 = 2$, we have the eigenvector $k \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

For the eigenvalue $\lambda_2 = 7$, we have the eigenvector $k \begin{bmatrix} \frac{1}{4} \\ 1 \end{bmatrix}$

(b) $\lambda = 0, 2, 7$.

For the eigenvalue $\lambda_1 = 0$, we have the eigenvector $k \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$.

For the eigenvalue $\lambda_2 = 2$, we have the eigenvector $k \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

For the eigenvalue $\lambda_3 = 7$, we have the eigenvector $k \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$.

(c) $\lambda_1 = 1$; $\lambda_2 = -1$; $\lambda_3 = -1$ are the eigenvalues.

The eigenvector corresponding to $\lambda_1 = 1$ is $k \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

The eigenvector corresponding to $\lambda_2 = \lambda_3 = -1$ is $k \begin{bmatrix} -\frac{1}{2} \\ 0 \\ 1 \end{bmatrix}$.

19. $\lambda = 5$ is an eigenvalue, and $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ or more generally, $\begin{bmatrix} k \\ k \\ k \end{bmatrix}$, is the corresponding eigenvector.

20. $\lambda = 0$ is an eigenvalue.