MTH #3331 Practice Test #3

SUMMER 2013

Pat Rossi Name ________________________________

1. Given vectors $\mathbf{u} = (2,3)$ and $\mathbf{v} = (1,1)$, compute:

   (a) $\mathbf{proj_v}(\mathbf{u})$

   (b) $\mathbf{orth_v}(\mathbf{u})$

2. Calculate $\det(A)$ for:

   (a) $A = \begin{bmatrix} 1 & -1 & 2 & -4 \\ 1 & 0 & -4 & 2 \\ 1 & -1 & 0 & 0 \\ 2 & 2 & -2 & 6 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & 2 & -4 \\ 2 & 1 & 0 & -4 & 2 \\ 3 & 1 & -1 & 0 & 0 \\ 4 & 2 & 2 & -2 & 6 \end{bmatrix}$

3. Calculate $\det(A)$ for:

   (a) $A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 0 & 0 \\ 1 & 5 & 7 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ -1 & 4 & 0 \end{bmatrix}$

   (d) $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

   (e) $A = \begin{bmatrix} 3 & 1 & 4 \\ 3 & 1 & 4 \\ 1 & 5 & 7 \end{bmatrix}$

   (f) $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & -2 & 3 \\ 2 & -4 & 5 \end{bmatrix}$

4. If A and B are matrices, and $\det(A) = \det(B)$, is it necessarily true that $A = B$?

5. Write as a sum of two determinants, and compute:

   (a) $\det\left(\begin{bmatrix} 1-k & 2+0 & 3+0 \\ 4 & 1 & 7 \\ 2 & 8 & 9 \end{bmatrix}\right)$

   (b) $\det\left(\begin{bmatrix} 1-k & 4-2 & 1+2 \\ 4 & 1 & 7 \\ 2 & 8 & 9 \end{bmatrix}\right)$

6. Compute the determinant of $A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 2 \\ 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}$

7. Compute the determinants using cofactors:

   (a) $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 2 & 1 & 5 & 2 \\ 3 & 0 & 0 & 1 \\ 4 & 0 & 0 & 1 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 3 & 1 & 4 & 2 \\ 0 & 5 & 0 & 7 \\ 1 & -1 & 1 & -1 \end{bmatrix}$

8. Combine the method of row reduction and cofactors to calculate $\det(A)$, if: $A = \begin{bmatrix} 1 & 1 & 3 & 1 \\ 2 & 1 & 4 & 2 \\ 1 & -1 & 5 & 7 \\ -2 & 1 & 1 & 3 \end{bmatrix}$

9. Calculate $\det(A)$, $\det(B)$, and $\det(AB)$ if:

$$A = \begin{bmatrix} 1 & 0 & 1 & 5 \\ 2 & 0 & -1 & 3 \\ 1 & 2 & 0 & 2 \\ -1 & 1 & 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 2 \\ 1 & 4 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

10. Characterize the following matrices as singular or non-singular. Justify your answers.

(a) $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 1 \\ -3 & 0 & 6 \\ -2 & 2 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 4 & 1 \\ 7 & 0 & 0 & 0 \\ 6 & -1 & 1 & 0 \\ 11 & 0 & 0 & 4 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 3 & 6 & 7 & 1 \end{bmatrix}$

11. Which of the following matrices are singular?

(a) $\begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

12. Given that A is a 6×6 matrix, and that $\det(A) = 5$, what is the rank of A?

13. The system of equations $A\mathbf{x} = \mathbf{b}$ is a system of five equations and five unknowns. It has a unique solution. What is the rank of A?

14. A is the matrix $\begin{bmatrix} 1 & a & 0 & 0 \\ a & 1 & 0 & 0 \\ 1 & a & 1 & a \\ 1 & -1 & a & 1 \end{bmatrix}$. Show that if the system of equations $A\mathbf{x} = \mathbf{0}$ has more than one solution, then we must have: $a = 1$ or $a = -1$.

15. If A and B are $n \times n$ matrices with rank n, show that AB has rank n.

16. If A and B are $n \times n$ matrices with rank less than n, show that AB has rank less than n.

17. For what value(s) of λ does $A\mathbf{x} = \lambda\mathbf{x}$ have a nontrivial solution, if $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$?

    Find the vectors $\mathbf{x}$ associated with each λ.

18. Find the eigenvalues and eigenvectors of A:

    (a) $\begin{bmatrix} 3 & 1 \\ 4 & 6 \end{bmatrix}$

    (b) $\begin{bmatrix} 7 & 7 & 7 \\ -5 & -7 & -9 \\ 5 & 7 & 9 \end{bmatrix}$

    (c) $\begin{bmatrix} 3 & -1 & 2 \\ 0 & -1 & 0 \\ -4 & 2 & -3 \end{bmatrix}$

19. Given that $\begin{bmatrix} 4 & -2 & 3 \\ 1 & 3 & 1 \\ 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$, state one eigenvalue of the matrix $\begin{bmatrix} 4 & -2 & 3 \\ 1 & 3 & 1 \\ 3 & 2 & 0 \end{bmatrix}$, and its corresponding eigenvector.

20. Calculation will show that $\det\left(\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}\right) = 0$, what does this information tell us about the eigenvalues of the matrix?