MTH #3331 Practice Test #4 - Answers

Summer 2013

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1. ~

   (a)
   1. A is non-singular.
   2. A is row equivalent to I.
   3. A has rank n.
   4. A is invertible (i.e., A^{-1} exists).
   5. The system of equations $A\mathbf{x} = \mathbf{b}$ has a unique solution.
   6. The system of equations $A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\mathbf{x} = \mathbf{0}$.
   7. The rows of A are linearly independent.
   8. The columns of A are linearly independent.
   9. $\det(A) \neq 0$.
   10. The rows of A span $\Re^n$.
   11. The columns of A span $\Re^n$.

2. ~

   (a)
   1. A is singular.
   2. A is **not** row equivalent to I.
   3. A has rank less than n.
   4. A is **not** invertible (i.e., A^{-1} **does not** exist).
   5. The system of equations $A\mathbf{x} = \mathbf{b}$ is either inconsistent or has infinitely many solutions.
   6. The system of equations $A\mathbf{x} = \mathbf{0}$ has a non-trivial solution.
   7. The rows of A are linearly dependent.
   8. The columns of A are linearly dependent.
   9. $\det(A) = 0$.
   10. The rows of A **do not** span $\Re^n$.
   11. The columns of A **do not** span $\Re^n$.

3. A **basis** of $\Re^n$ is a set of vectors of $\Re^n$ which is linearly independent, and spans of $\Re^n$.

   Alternatetively, a **basis** of $\Re^n$ is a set of vectors of $\Re^n$ which has the property that every vector in $\Re^n$ can be expressed **uniquely** as a linear combination of the elements of the set.

4. A set of n-tuples of $\Re^n$ is said to **span** $\Re^n$ if every vector in $\Re^n$ can be written as a linear combination of the n-tuples of the set.

5. ~

   (a) ~

      1. Since the set does not have exactly n elements, the set cannot be a basis of $\Re^n$.
      2. Since the set has less than n elements, it fails to span $\Re^n$.

   (b) ~

      1. Since the set does not have exactly n elements, the set cannot be a basis of $\Re^n$.
      2. Since the set has more than n elements, the elements of the set are linearly dependent.

6. A set of n-tuples can fail to be a basis of $\Re^n$ by either

   (a) not spanning of $\Re^n$, or
   (b) not being linearly independent (or both).

7. See solutions.

8. ~

   (a) Linearly dependent.
   (b) Linearly independent.
   (c) Linearly independent.
   (d) Linearly dependent.
   (e) Linearly dependent.

9. See Solutions.

10. See Solutions.

11. See Solutions.

12. ~

   (a) A^{-1} does not exist.
   (b) A^{-1} exists.
   (c) A^{-1} does not exist.
   (d) A^{-1} does not exist.
   (e) A^{-1} does not exist.
   (f) A^{-1} exists.

13. ~

   (a) $\mathbf{B}'$ is not a basis of $\Re^n$. $\mathbf{B}'$ is linearly independent also. Finally, $\mathbf{B}'$ does not span $\Re^n$.
   (b) $\mathbf{B}'$ is not a basis of $\Re^n$. $\mathbf{B}'$ cannot span $\Re^n$. $\mathbf{B}'$ may or may not be linearly independent.
   (c) $\mathbf{B}'$ is not a basis of $\Re^n$. $\mathbf{B}'$ is linearly dependent. $\mathbf{B}'$ must span $\Re^n$ also.

14. $\mathbf{B}$ is not a basis of $\Re^n$. Alternately, $\mathbf{B}$ is linearly dependent.

15. ~

   (a) Yes.
   (b) No.
   (c) No.
   (d) Yes.
   (e) Yes.
   (f) No.
   (g) No.
   (h) No.

16. ~

   (a) Not a basis.
   (b) This is a basis.
   (c) Not a basis.
   (d) Not a basis.
   (e) Not a basis.

17. $\begin{bmatrix} 1 & 0 & -5 & 26 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ rank is 2.

18. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 0 \\ 0 \end{bmatrix}$

19. $a = -1$; $b = 0$