MTH #3331 Practice Test #4

SUMMER 2013

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1. Complete the following statement: The following statements are equivalent for an $n \times n$ matrix A :

   (a) 1. A is non-singular.
       2. ~
       3. ~
       4. ~
       $\vdots$

2. Complete the following statement: The following statements are equivalent for an $n \times n$ matrix A :

   (a) 1. A is singular.
       2. ~
       3. ~
       4. ~
       $\vdots$

3. Define a **basis** of $\Re^n$.

4. Define what it means for a set of n-tuples to **span** $\Re^n$.

5. What can be said about:

   (a) a set of n-tuples in $\Re^n$ having less than n elements?

   (b) a set of n-tuples in $\Re^n$ having more than n elements?

6. How can a set of n-tuples fail to be a basis of $\Re^n$?

7. Given the set of 3-tuples $\{(1,1,0),(0,1,1),(1,0,1),(1,1,1)\}$, show that the fourth 3-tuple is a linear combination of the first three.

8. Decide by inspection whether each set is linearly independent or dependent. Give a reason for your answer.

   (a) $\{(1,1,2),(1,4,5),(1,2,7),(-1,8,3)\}$

   (b) $\{(1,1,0,0),(0,0,1,1)\}$

   (c) $\{(1,0,0,0),(1,1,0,0),(1,1,1,0)\}$

   (d) The columns of the matrix $\begin{bmatrix} 1 & 5 & 4 \\ 2 & 8 & -3 \end{bmatrix}$

   (e) $\{(0,0,0),(1,1,5),(2,8,7)\}$

9. Show that any set containing the zero vector, $\mathbf{0}$, is linearly dependent.

10. Show that if the set $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$ is linearly independent, then any subset of this set is also linearly independent.

11. Show that if the set $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$ is linearly dependent, then any set containing these as a subset is also linearly dependent.

12. By inspection, determine whether A^{-1} exists for each of the following. (Give a reason for your answer.)

    (a) $\begin{bmatrix} 2 & 0 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

    (b) $\begin{bmatrix} 2 & 0 & 0 \\ 3 & 4 & 0 \\ 5 & 6 & 7 \end{bmatrix}$

    (c) $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

    (d) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

    (e) $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

    (f) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

13. Let $\mathbf{B} = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n\}$ be a basis for $\Re^n$. What can be said about the following? (Give more of a reason for your answer than $\mathbf{B}'$ is a basis or $\mathbf{B}'$ is not a basis.)

    (a) $\mathbf{B}' = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_{n-1}\}$

    (b) $\mathbf{B}' = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_{n-2}, \mathbf{x}\}$ where $\mathbf{x}$ is a vector in $\Re^n$.

    (c) $\mathbf{B}' = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_{n-1}, \mathbf{b}_n, \mathbf{x}\}$ where $\mathbf{x}$ is a vector in $\Re^n$.

14. Let $\mathbf{B} = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_m\}$ be a set of vectors in $\Re^n$. Suppose also that there are elements $\mathbf{s}_1$ and $\mathbf{s}_2$ in $\Re^n$ such that $\mathbf{s}_1 = c_1\mathbf{b}_1 + c_2\mathbf{b}_2 + \ldots + c_m\mathbf{b}_m$ and $\mathbf{s}_2 = k_1\mathbf{b}_1 + k_2\mathbf{b}_2 + \ldots + k_m\mathbf{b}_m$ with scalars $c_i \neq k_i$ for $i = 1, 2, \ldots, n$. What can we say about $\mathbf{B}$ and why, if $\mathbf{s}_1 - \mathbf{s}_2 = 0$?

15. Which of the following could be a basis for $\Re^3$? Explain why or why not (e.g., if you claim linear independence or dependence, give a reason why we know the set is linearly independent or dependent.)

(a) $\mathbf{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

(b) $\mathbf{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

(c) $\mathbf{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

(d) $\mathbf{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \right\}$

(e) $\mathbf{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

(f) $\mathbf{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

(g) $\mathbf{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ -4 \\ 3 \end{bmatrix} \right\}$

(h) $\mathbf{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

16. Determine, by inspection, which of the following sets of 4-tuples are bases for $\Re^4$. Give a reason in each case.

(a) $\mathbf{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

(b) $\mathbf{B} = \left\{ \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \right\}$

(c) $\mathbf{B} = \left\{ \begin{bmatrix} 1 \\ 6 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 9 \\ 6 \end{bmatrix} \right\}$

(d) $\mathbf{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

(e) $\mathbf{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

17. Write in reduced (row echelon) form and determine the rank.

$$A = \begin{bmatrix} 1 & 4 & 3 & 6 \\ 2 & 9 & 8 & 7 \\ 6 & 26 & 22 & 26 \\ 5 & 22 & 19 & 20 \end{bmatrix}$$

18. Solve the system of equations by transforming $\left[A \,|\, \vec{b}\right] \Rightarrow \left[I \,|\, \vec{b}'\right]$.

$$\begin{array}{rrrrl} x_1 & +2x_2 & +x_3 & +x_4 & = 2 \\ x_1 & -x_2 & +4x_3 & -4x_4 & = -4 \\ 2x_1 & +x_2 & +5x_3 & +5x_4 & = -2 \\ 3x_1 & & -9x_3 & -9x_4 & = -6 \end{array}$$

19. Let $\vec{u}_1 = (1, 2, 1, 3)$, $\vec{u}_2 = (1, -1, 2, 0)$, $\vec{u}_3 = (1, a, 2, b)$. Choose a and b so that the set $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ is linearly dependent.