MTH 3331 Test #1 - Solutions

SUMMER 2013

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Instructions. Show clearly how you arrive at your answers.

1. Given

$$A = \begin{bmatrix} 2 & 1 & \frac{1}{2} \\ \frac{1}{2} & 1 & 0 \\ 3 & 1 & \frac{1}{2} \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & -\frac{1}{2} \\ 5 & -2 & 3 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 0 & 4 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad E = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \qquad F = \begin{bmatrix} -2 & -1 & -\frac{1}{2} \\ -\frac{1}{2} & -1 & 0 \\ -3 & -1 & -\frac{1}{2} \end{bmatrix}$$

Without actually computing the following sums and products, indicate which of these can be computed and which can't.

(a) AB Yes.

(b) BC Yes.

(c) CB No. (# of columns of C not equal to # of rows of B)

(d) $C+B$ No. (Matrices are not the same size.)

(e) $A+B$ Yes.

(f) CD Yes.

(g) DC No. (# of columns of D not equal to # of rows of C)

(h) $C+D$ No. (Matrices are not the same size.)

2. $A = \begin{bmatrix} 2 & 1 & \frac{1}{2} \\ \frac{1}{2} & 1 & 0 \\ 3 & 1 & \frac{1}{2} \end{bmatrix}$ $\quad B = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & -\frac{1}{2} \\ 5 & -2 & 3 \end{bmatrix}$ $\quad F = \begin{bmatrix} -2 & -1 & -\frac{1}{2} \\ -\frac{1}{2} & -1 & 0 \\ -3 & -1 & -\frac{1}{2} \end{bmatrix}$

(a) Compute AB.

$$AB = \begin{bmatrix} 2 & 1 & \frac{1}{2} \\ \frac{1}{2} & 1 & 0 \\ 3 & 1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & -\frac{1}{2} \\ 5 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (2)(-1) + (1)(2) + \left(\frac{1}{2}\right)(5) & (2)(0) + (1)(1) + \left(\frac{1}{2}\right)(-2) & (2)(1) + (1)\left(-\frac{1}{2}\right) + \left(\frac{1}{2}\right)(3) \\ \left(\frac{1}{2}\right)(-1) + (1)(2) + (0)(5) & \left(\frac{1}{2}\right)(0) + (1)(1) + (0)(-2) & \left(\frac{1}{2}\right)(1) + (1)\left(-\frac{1}{2}\right) + (0)(3) \\ (3)(-1) + (1)(2) + \left(\frac{1}{2}\right)(5) & (3)(0) + (1)(1) + \left(\frac{1}{2}\right)(-2) & (3)(1) + (1)\left(-\frac{1}{2}\right) + \left(\frac{1}{2}\right)(3) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{2} & 0 & 3 \\ \frac{3}{2} & 1 & 0 \\ \frac{3}{2} & 0 & 4 \end{bmatrix}$$

(b) Compute $A + F$.

$$A+F = \begin{bmatrix} 2 & 1 & \frac{1}{2} \\ \frac{1}{2} & 1 & 0 \\ 3 & 1 & \frac{1}{2} \end{bmatrix} + \begin{bmatrix} -2 & -1 & -\frac{1}{2} \\ -\frac{1}{2} & -1 & 0 \\ -3 & -1 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 + (-2) & 1 + (-1) & \frac{1}{2} + \left(-\frac{1}{2}\right) \\ \frac{1}{2} + \left(-\frac{1}{2}\right) & 1 + (-1) & 0 + 0 \\ 3 + (-3) & 1 + (-1) & \frac{1}{2} + \left(-\frac{1}{2}\right) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

3. Referring to the matrices in problem 2, compute $BA + BF$. (Hint: Think about this for a second.)

$$BA + BF = B(A + F) = B \cdot 0_{3\times 3} = 0_{3\times 3}$$

4. Write the system of equations

$$\begin{array}{rrrcl} u & +v & -7w & = & 20 \\ 3u & +9v & +7w & = & 20 \\ 4u & +4v & +4w & = & 10 \end{array}$$

in the form: $\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \\ 10 \end{bmatrix}$

$$\begin{array}{rrrcl} u & +v & -7w & = & 20 \\ 3u & +9v & +7w & = & 20 \\ 4u & +4v & +4w & = & 10 \end{array} = \begin{bmatrix} 1 & 1 & -7 \\ 3 & 9 & 7 \\ 4 & 4 & 4 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \\ 10 \end{bmatrix}$$

5. For $(n \times n)$ matrices A and B, compute $(A + B)(A - B)$, simplifying to the extent possible.

$$(A + B)(A - B) = (A + B)A - (A + B)B = AA + BA - AB - BB$$

$$= A^2 + BA - AB - B^2$$

i.e., $(A + B)(A - B) = A^2 + BA - AB - B^2$

ALTERNATIVELY:

$$(A + B)(A - B) = A(A - B) + B(A - B) = AA - AB + BA - BB$$

$$= A^2 - AB + BA - B^2$$

i.e., $(A + B)(A - B) = A^2 - AB + BA - B^2$

6. For

$$A = \begin{bmatrix} 7 & 6 & 3 \\ 2 & 1 & 0 \\ 9 & 8 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

compute $3A + 2B + C$.

$$3A + 2B + C = 3\begin{bmatrix} 7 & 6 & 3 \\ 2 & 1 & 0 \\ 9 & 8 & 5 \end{bmatrix} + 2\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (3)(7) & (3)(6) & (3)(3) \\ (3)(2) & (3)(1) & (3)(0) \\ (3)(9) & (3)(8) & (3)(5) \end{bmatrix} + \begin{bmatrix} (2)(-1) & (2)(1) & (2)(1) \\ (2)(1) & (2)(-1) & (2)(1) \\ (2)(2) & (2)(3) & (2)(4) \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 18 & 9 \\ 6 & 3 & 0 \\ 27 & 24 & 15 \end{bmatrix} + \begin{bmatrix} -2 & 2 & 2 \\ 2 & -2 & 2 \\ 4 & 6 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 + (-2) + 1 & 18 + 2 + 1 & 9 + 2 + 0 \\ 6 + 2 + 2 & 3 + (-2) + 1 & 0 + 2 + 0 \\ 27 + 4 + 1 & 24 + 6 + 1 & 15 + 8 + 1 \end{bmatrix} = \begin{bmatrix} 20 & 21 & 11 \\ 10 & 2 & 2 \\ 32 & 31 & 24 \end{bmatrix}$$

7. Given

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 2 \\ 2 & -1 \end{bmatrix} \qquad C = \begin{bmatrix} 5 & -2 \\ 1 & 1 \end{bmatrix}$$

(a) Compute AB and AC.

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} (1)(3) + (2)(2) & (1)(2) + (2)(-1) \\ (3)(3) + (6)(2) & (3)(2) + (6)(-1) \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 21 & 0 \end{bmatrix}$$

$$AC = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} (1)(5) + (2)(1) & (1)(-2) + (2)(1) \\ (3)(5) + (6)(1) & (3)(-2) + (6)(1) \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 21 & 0 \end{bmatrix}$$

(b) What does the result in problem 7.a tell us about matrix multiplication?

The "cancelation laws" do not hold. (i.e., $AB = AC \nRightarrow B = C$.)

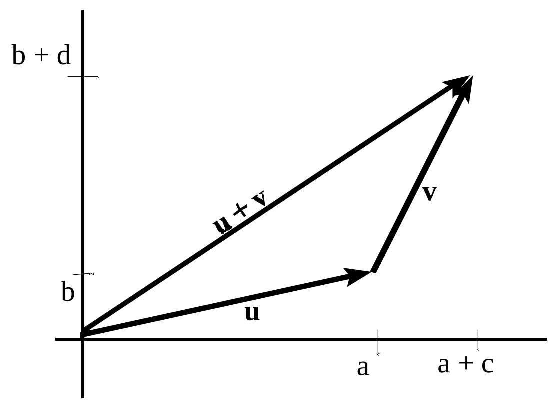
8. Simplify to the extent possible: $\left[C^T \cdot \left(A^T + B^T\right)\right]^T =$

$$\left[C^T \cdot \left(A^T + B^T\right)\right]^T = \left(A^T + B^T\right)^T \cdot \left(C^T\right)^T = \left(\left(A^T\right)^T + \left(B^T\right)^T\right) \cdot \left(C^T\right)^T$$

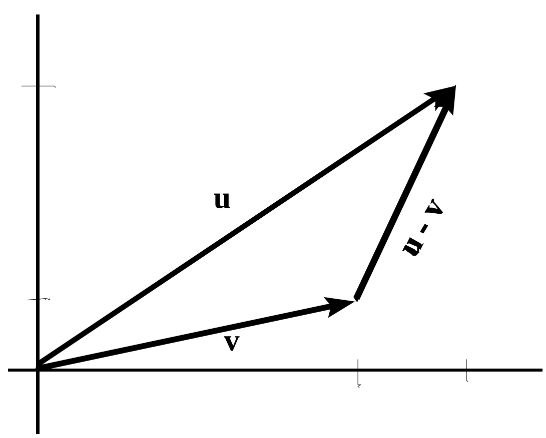
$$= (A + B) \cdot C = AC + BC$$

9. Given the vectors $\mathbf{u} = (a, b)$ and $\mathbf{v} = (c, d)$, give the geometric interpretation of the following:

(a) $\tilde{\mathbf{u}} + \tilde{\mathbf{v}}$



(b) $\tilde{\mathbf{u}} - \tilde{\mathbf{v}}$



ALTERNATELY: