MTH 3331 Test #1

SPRING 2018

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Instructions. Show clearly how you arrive at your answers.

1. Write the system of equations

$$\begin{array}{rcrcrcr} 2x & + & 3y & - & 8z & = & 3 \\ -2x & + & y & + & 4z & = & 7 \\ 8x & + & 3y & + & 2z & = & 2 \end{array}$$

in the form: $\left[\quad A \quad \right] \left[\vec{x} \right] = \left[\vec{b} \right]$

2. For $(n \times n)$ matrices A and B, compute $(A - B)\left(A^2 + AB + B^2\right)$, simplifying to the extent possible.

3. If matrices A, B, C are such that A is nonzero and $AB = AC$, what can we conclude?

4. If matrices A and B are $n \times n$ matrices, and $AB = 0_{n \times n}$, what can we conclude?

5. Define Diagonal Matrix.

6. What is the multiplicative identity for $n \times n$ matrices, and what property does it have?

7. Given the vectors $\tilde{\mathbf{u}} = (a, b)$ and $\tilde{\mathbf{v}} = (c, d)$, give the geometric interpretation of the following:

(a) $\tilde{\mathbf{u}} + \tilde{\mathbf{v}}$

(b) $\tilde{\mathbf{u}} - \tilde{\mathbf{v}}$

8. For

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \qquad C = \begin{bmatrix} 7 & 6 & 3 \\ 2 & 1 & 0 \\ 9 & 8 & 5 \end{bmatrix}$$

compute $3A + 4B - 2C$.

9. For $n \times n$ matrices A, B, C, write the transpose of each matrix

(a) A^T

(b) $A^T A$

(c) $A(B + C)$

(d) ABC^T

(e) $AB^T A^T$

(f) $A^T B + B^T A$

10. For the matrices given below, calculate each of the following products when they are defined.

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 4 & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

(a) AB

(b) AC

(c) CD

(d) BC

(e) DC

(f) $BC + CD$

(g) $AC + BC$

(h) $(A + B)\,C$

11. Given that $\tilde{\mathbf{u}} = (1, 2, 3, 4)$ and $\tilde{\mathbf{v}} = (3, 1, 2, 1)$, compute the dot product $\mathbf{u} \circ \mathbf{v}$