MTH 3331 Test #1 - Solutions

Summer 2015

Pat Rossi Name ______________________

Instructions. Show clearly how you arrive at your answers.

1. Write the system of equations

$$\begin{array}{rrrcl} 4x & -7y & +3z & = & 9 \\ -2x & & +z & = & 5 \\ 8x & +3y & +2z & = & 2 \end{array}$$

in the form: $\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 2 \end{bmatrix}$

2. For $(n \times n)$ matrices A and B, compute $(A - B)(A^2 + AB + B^2)$, and simplify to the extent possible.

3. If matrices A, B, C are such that A is nonzero and $AB = AC$, what can we conclude?

4. Define Diagonal Matrix.

5. What is the multiplicative identity for $n \times n$ matrices, and what special property does it have?

6. For

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \qquad C = \begin{bmatrix} 7 & 6 & 3 \\ 2 & 1 & 0 \\ 9 & 8 & 5 \end{bmatrix}$$

compute $4A + 2B - C$.

7. Given

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix}$$

(a) Compute AB

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(b) What does the result in problem 7.a tell us about matrix multiplication?

8. Given the vectors $\tilde{\mathbf{u}} = (a, b)$ and $\tilde{\mathbf{v}} = (c, d)$, give the geometric interpretation of the following:

(a) $\tilde{\mathbf{u}} + \tilde{\mathbf{v}}$

(b) $\tilde{\mathbf{u}} - \tilde{\mathbf{v}}$

9. For the matrices given below, calculate each of the following products when they are defined.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 0 & 4 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

(a) AB

(b) AC

(c) CD

(d) BC

(e) DC

(f) $BC + CD$

(g) $(A + B)C$

(h) $AC + BC$

10. State the Cauchy-Schwarz Inequality.

11. State the Triangle Inequality and give its geometric interpretation.

12. Given that $\tilde{\mathbf{u}} = (1, 2, 3, 4)$ and $\tilde{\mathbf{v}} = (3, 1, 2, 1)$, compute the dot product $\mathbf{u} \circ \mathbf{v}$.

13. Find $\cos(\theta)$, where θ is the angle between vectors $\tilde{\mathbf{u}} = (2, 4, 1)$ and $\tilde{\mathbf{v}} = (-2, 2, -4)$

14. Find the norm or "length" of the vectors $\tilde{\mathbf{u}} = (3, 4, 12)$ and $\tilde{\mathbf{v}} = (4, 4, 5, -2)$

15. For $n \times n$ matrices A, B, C, write the transpose of each matrix

    (a) A^T

    (b) AA^T

    (c) $A(B + C)$

    (d) ABC

    (e) $A^T BA$

    (f) $A^T B + B^T A$

16. ~

    (a) Show that $||k\mathbf{v}|| = |k|\,||\mathbf{v}||$. (Hint: Somewhere along the line, you'll have to use the fact that the absolute value of a product equals the product of the absolute values. i.e., $|ab| = |a|\,|b|$.

17. ~

    (a) Find a unit vector that has the same direction as the vector $(-1, 4, 3)$.

    (b) Find a unit vector that has the opposite direction of the vector $(0, 2, 5)$.

18. Verify the Cauchy-Schwartz Inequality in the following case:

    (a) $\mathbf{u} = (1, -1, 3, 4)$ and $\mathbf{v} = (2, 0, 3, 1)$.

19. Let $\mathbf{u} = (1, 1, 2)$ and $\mathbf{v} = (k, 0, 3)$. Choose k such that:

    (a) $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.

    (b) the angle between $\mathbf{u}$ and $\mathbf{v}$ is $\frac{\pi}{6}$.