MTH 3331 - Test #2

SUMMER 2013

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Instructions. Show clearly how you arrive at your answers.

1. State the Cauchy-Schwarz Inequality.

2. State the Triangle Inequality and give its geometric interpretation.

3. Find the norm or "length" of the vectors $\tilde{\mathbf{u}} = (3, 4, 12)$ and $\tilde{\mathbf{v}} = (4, 4, 5, -2)$

4. Given that $\tilde{\mathbf{u}} = (1, 2, 3, 4)$ and $\tilde{\mathbf{v}} = (3, 1, 2, 1)$, compute the dot product $\mathbf{u} \circ \mathbf{v}$.

5. Find $\cos(\theta)$, where θ is the angle between vectors $\tilde{\mathbf{u}} = (2, 4, 1)$ and $\tilde{\mathbf{v}} = (-2, 2, -4)$

6. Find a unit vector that has the same direction as the vector $(-1, 4, 3)$.

7. Let $\mathbf{u} = (1, 1, 2)$ and $\mathbf{v} = (k, 0, 3)$. Choose k such that: $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.

8. Solve the system of equations below:

$$\left[\begin{array}{rcrcrcl} x_1 & + & x_2 & + & 2x_3 & = & 9 \\ 2x_1 & + & 4x_2 & - & 3x_3 & = & 1 \\ 3x_1 & + & 6x_2 & - & 5x_3 & = & 0 \end{array}\right]$$

9. Compute A^{-1} if

$$A = \begin{bmatrix} 3 & 2 & -1 \\ -2 & 6 & \frac{7}{6} \\ -1 & -1 & \frac{1}{3} \end{bmatrix}$$

10. Solve the systems below, in the most economical way possible:

(a)
$$\begin{aligned} 3x_1 + 2x_2 - x_3 &= 20 \\ -2x_1 + 6x_2 + \tfrac{7}{6}x_3 &= 5 \\ -x_1 - x_2 + \tfrac{1}{3}x_3 &= 5 \end{aligned}$$

(b)
$$\begin{aligned} 3x_1 + 2x_2 - x_3 &= 10 \\ -2x_1 + 6x_2 + \tfrac{7}{6}x_3 &= 10 \\ -x_1 - x_2 + \tfrac{1}{3}x_3 &= 10 \end{aligned}$$

(c)
$$\begin{aligned} 3x_1 + 2x_2 - x_3 &= -2 \\ -2x_1 + 6x_2 + \tfrac{7}{6}x_3 &= 2 \\ -x_1 - x_2 + \tfrac{1}{3}x_3 &= 6 \end{aligned}$$

11. Determine the equation of the plane which contains the point $(3, -1, 2)$, and which has normal vector $\langle 1, -1, 5 \rangle$.

12. Find the parametric equations of the line which contains the points $(3, 1, -2)$ and $(1, -1, 5)$.

13. Solve the system of equations by reducing the augmented matrix to row-reduced echelon form.

$$\begin{array}{rcrcrcrcr} x_1 & + & 2x_2 & + & x_3 & + & x_4 & = & 2 \\ x_1 & - & x_2 & + & 4x_3 & + & 4x_4 & = & -4 \\ 2x_1 & + & x_2 & + & 5x_3 & + & 5x_4 & = & -2 \\ 3x_1 & & & + & 9x_3 & + & 9x_4 & = & -6 \end{array}$$

14. Given the system

$$\begin{bmatrix} 1 & 4 & 2 \\ 2 & 5 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}$$

the row-reduced form of the augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Write the solution set of this system as a linear combination of 3-tuples.