Induction Problems - Assignment #1

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Instructions. Prove the following by Mathematical Induction:

1. $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

   i.e. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

2. $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

   i.e. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

3. $1 + x + x^2 + x^3 + \ldots + x^n = \frac{x^{n+1}-1}{x-1}$; where $x \neq 1$.

   i.e. $\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$; where $x \neq 1$.

4. Given that $|x_1 + x_2| \leq |x_1| + |x_2|$ (the Triangle Inequality); Prove by induction that:

   $|x_1 + x_2 + x_3 + \ldots + x_n| \leq |x_1| + |x_2| + |x_3| + \ldots + |x_n|$ (the General Triangle Inequality).

5. $(1 + x)^n \geq 1 + nx$ for any natural number n and any real number $x \geq -1$.

6. For $0 \leq a \leq b$; prove that $a^n \leq b^n$.

7. $1 + 3 + 5 + \ldots + (2n - 1) = n^2$

   i.e. $\sum_{i=1}^{n} (2i - 1) = n^2$

8. $2 + 4 + 6 + \ldots + 2n = n^2 + n$

   i.e. $\sum_{i=1}^{n} 2i = n^2 + n$