Hint for the Power Set Proof

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Hint for the Power Set Problem

For ease of notation, let $n(A)$ denote the number of elements in set A and let $n(P(A))$ denote the number of elements in $P(A)$ (the power set of A), etc.

i.e., Let $n(A)$ denote the cardinality of set A and let $n(P(A))$ denote the cardinality of $P(A)$ (the power set of A), etc.

To prove, by induction, that:

$$\text{If } n(A) = n, \text{ then } n(P(A)) = 2^n,$$

do the following:

First, prove our proposition for $n = 0$.

Next, assume that the proposition is true for $n = k$ and show that the proposition is true for $n = k + 1$.

i.e., assume that $\underbrace{\text{if } n(A) = k, \text{ then } n(P(A)) = 2^k}_{\text{induction hypothesis}}$,

and show that if $n(A) = k + 1$, then $n(P(A)) = 2^{k+1}$.

Now, the big question: How do we DO this?

Without loss of generality, assume that the set having k elements is

$$A_k = \{a_1, a_2, a_3, \ldots, a_k\}$$

and that the set having $k + 1$ elements is

$$A_{k+1} = \{a_1, a_2, a_3, \ldots, a_k, a_{k+1}\}$$

Next observe that every subset of A_k is also a subset of A_{k+1}.

Hence, every element of $P(A_k)$ is also an element of $P(A_{k+1})$.

Finally, consider the subsets of A_{k+1} that are not subsets of A_k, and ask yourself: "What do these subsets look like?" or "Where do these subsets come from?"