

MTH 4441 Homework - Groups and Group Axioms

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Name _____

Decide whether each of the given sets is a group with respect to the given operation. If it is NOT a group, state one of the group axioms that fails to hold.

1. The set \mathbb{Z}^+ of all positive integers with operation addition.
2. The set \mathbb{Z}^+ of all positive integers with operation multiplication.
3. The set \mathbb{Q} of all rational numbers with operation addition.
4. The set \mathbb{Q}' of all irrational numbers with operation addition.
5. The set of all positive irrational numbers with operation multiplication.
6. The set \mathbb{Q}^+ of all positive rational numbers with operation multiplication.
7. The set of all real numbers x such that $0 < x \leq 1$ with operation multiplication.
8. The set \mathbf{E} of all even integers with operation addition.
9. The set \mathbf{E} of all even integers with operation multiplication.
10. The set of all multiples of a positive integer n with operation addition.
11. The set of all multiples of a positive integer n with operation multiplication.

In Exercises 12-13, the given table defines an operation of multiplication on the set $S = \{e, a, b, c\}$. In each case, find a group axiom that fails to hold, and thereby show that S is not a group.

12.

\times	e	a	b	c
e	e	a	b	c
a	a	b	a	b
b	b	c	b	c
c	c	e	c	e

13.

\times	e	a	b	c
e	e	a	b	c
a	e	a	b	c
b	e	a	b	c
c	e	a	b	c