

MTH 4441 HW #3 - SUBGROUPS

FALL 2017

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Name _____

1. Given the group table for $(G, *)$, find all of the subgroups of $(G, *)$ and justify your answers. Draw a subgroup diagram for $(G, *)$.

*	e	a	b	c	d
e	e	a	b	c	d
a	a	b	c	d	e
b	b	c	d	e	a
c	c	d	e	a	b
d	d	e	a	b	c

2. Given the group table for $(G, *)$, find all of the subgroups of $(G, *)$ and justify your answers. Draw a subgroup diagram for $(G, *)$.

*	e	v	w	x	y	z
e	e	v	w	x	y	z
v	v	w	x	y	z	e
w	w	x	y	z	e	v
x	x	y	z	e	v	w
y	y	z	e	v	w	x
z	z	e	v	w	x	y

3. Given the group table for $(G, *)$, find all of the subgroups of $(G, *)$ and justify your answers. Draw a subgroup diagram for $(G, *)$.

*	e	v	w	x	y	z
e	e	v	w	x	y	z
v	v	e	x	z	w	y
w	w	x	e	y	z	v
x	x	z	y	e	v	w
y	y	w	z	v	e	x
z	z	y	v	w	x	e

4. Recall that $(\mathbb{Z}, +)$ is a group with identity 0, and that $(\{0\}, +)$ and $(\mathbb{Z}, +)$ must be subgroups of $(\mathbb{Z}, +)$.

Recall that $(2\mathbb{Z}, +)$, where $2\mathbb{Z} = \{0, \pm 2, \pm 4, \pm 6, \dots, \pm 2k, \dots\} = \{2k : k \in \mathbb{Z}\}$, is also a subgroup of $(\mathbb{Z}, +)$.

Does $(\mathbb{Z}, +)$ have any subgroups that are also subgroups of $(2\mathbb{Z}, +)$?

5. Recall that (\mathbb{Q}^+, \cdot) is a group with identity 1, and that $(\{1\}, \cdot)$ and (\mathbb{Q}^+, \cdot) must be subgroups of (\mathbb{Q}^+, \cdot) .

Recall that $(\{1, 2^{\pm 1}, 2^{\pm 2}, 2^{\pm 3}, \dots, 2^{\pm k}, \dots\}, \cdot)$, is also a subgroup of (\mathbb{Q}^+, \cdot) .

Does (\mathbb{Q}^+, \cdot) have any subgroups that are also subgroups of $(\{1, 2^{\pm 1}, 2^{\pm 2}, 2^{\pm 3}, \dots, 2^{\pm k}, \dots\}, \cdot)$?