MTH 4441 HW #3 - SUBGROUPS - Answers

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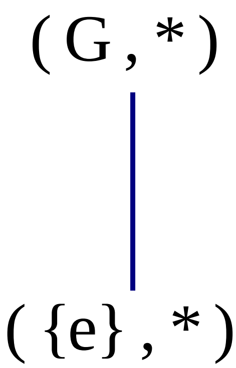
1. Given the group table for $(G, *)$, find all of the subgroups of $(G, *)$ and justify your answers. Draw a subgroup diagram for $(G, *)$.

$*$	e	a	b	c	d
e	e	a	b	c	d
a	a	b	c	d	e
b	b	c	d	e	a
c	c	d	e	a	b
d	d	e	a	b	c

$(\{e\}, *)$ and $(G, *)$ are the **only** subgroups of $(G, *)$.

Our subgroup diagram is below:

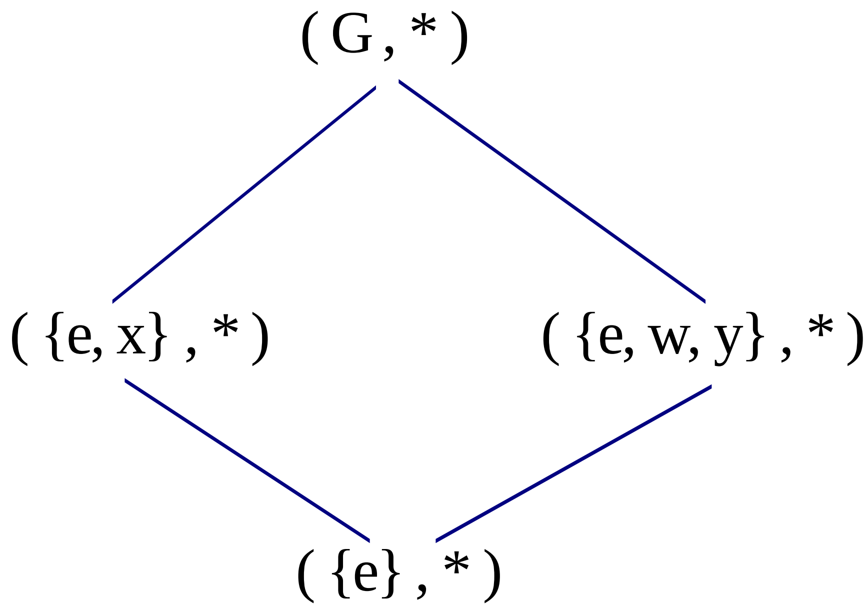


2. Given the group table for $(G, *)$, find all of the subgroups of $(G, *)$ and justify your answers. Draw a subgroup diagram for $(G, *)$.

$*$	e	v	w	x	y	z
e	e	v	w	x	y	z
v	v	w	x	y	z	e
w	w	x	y	z	e	v
x	x	y	z	e	v	w
y	y	z	e	v	w	x
z	z	e	v	w	x	y

$(\{e\}, *), (\{e, x\}, *), (\{e, w, y\}, *)$, and $(G, *)$ are the subgroups of $(G, *)$.

Our subgroup diagram is shown below:

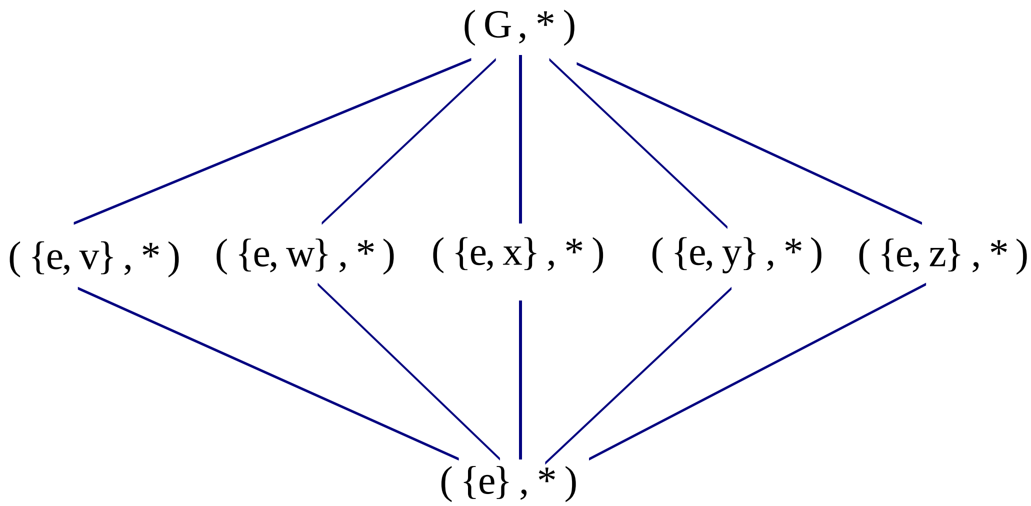


3. Given the group table for $(G,*)$, find all of the subgroups of $(G,*)$ and justify your answers. Draw a subgroup diagram for $(G,*)$.

$*$	e	v	w	x	y	z
e	e	v	w	x	y	z
v	v	e	x	z	w	y
w	w	x	e	y	z	v
x	x	z	y	e	v	w
y	y	w	z	v	e	x
z	z	y	v	w	x	e

The subgroups of $(G,*)$ are $(\{e\},*)$; $(\{e,v\},*)$; $(\{e,w\},*)$; $(\{e,x\},*)$; $(\{e,y\},*)$; $(\{e,z\},*)$; and $(G,*)$

Our subgroup diagram is shown below:



4. Recall that $(\mathbb{Z},+)$ is a group with identity 0, and that $(\{0\},+)$ and $(\mathbb{Z},+)$ must be subgroups of $(\mathbb{Z},+)$.

   Recall that $(2\mathbb{Z},+)$, where $2\mathbb{Z} = \{0, \pm 2, \pm 4, \pm 6, \ldots, \pm 2k, \ldots\} = \{2k : k \in \mathbb{Z}\}$, is also a subgroup of $(\mathbb{Z},+)$.

   Does $(\mathbb{Z},+)$ have any subgroups that are also subgroups of $(2\mathbb{Z},+)$?

   $(4\mathbb{Z},+)$; $(6\mathbb{Z},+)$; $(8\mathbb{Z},+)$; $(10\mathbb{Z},+)$; $(12\mathbb{Z},+)$; ... are subgroups of $(2\mathbb{Z},+)$.

5. Recall that $(\mathbb{Q}^+,\cdot)$ is a group with identity 1, and that $(\{1\},\cdot)$ and $(\mathbb{Q}^+,\cdot)$ must be subgroups of $(\mathbb{Q}^+,\cdot)$.

   Recall that $\left(\left\{1, 2^{\pm 1}, 2^{\pm 2}, 2^{\pm 3}, \ldots, 2^{\pm k}, \ldots\right\}, \cdot\right)$, is also a subgroup of $(\mathbb{Q}^+,\cdot)$.

   The subgroups of $\left(\left\{1, 2^{\pm 1}, 2^{\pm 2}, 2^{\pm 3}, \ldots, 2^{\pm k}, \ldots\right\}, \cdot\right)$ are

   $\left(\left\{1, 4^{\pm 1}, 4^{\pm 2}, 4^{\pm 3}, \ldots, 4^{\pm k}, \ldots\right\}, \cdot\right)$; $\left(\left\{1, 8^{\pm 1}, 8^{\pm 2}, 8^{\pm 3}, \ldots\right\}, \cdot\right)$; $\left(\left\{1, 16^{\pm 1}, 16^{\pm 2}, 16^{\pm 3}, \ldots\right\}, \cdot\right)$;

   $\left(\left\{1, 32^{\pm 1}, 32^{\pm 2}, 32^{\pm 3}, \ldots\right\}, \cdot\right)$; ...