# MTH 4441 HW - Working in $\mathbb{Z}_n$

Fall 2017

Pat Rossi Name ______________________________

1. Perform the following computations in $\mathbb{Z}_{12}$

   **Remark:** In each of the following exercises, our strategy will be to compute the value of each of the expressions, modulo 12, *using the proper remainders modulo 12.* The resulting value, modulo 12, will also be *proper remainder modulo 12.*

   a. $8 + 7$

   b. $10 + 9$

   c. $8 \cdot 11$

   d. $6 \cdot 9$

   e. $6 \cdot (9 + 11)$

   f. $5 \cdot (8 + 11)$

   g. $6 \cdot 9 + 6 \cdot 7$

   h. $5 \cdot 8 + 5 \cdot 11$

   i. $3 \cdot 7 + 4 \cdot 9$

   j. $8 \cdot 5 - 2 \cdot 10$

   k. $2^9$

   l. $3^4$

2. ~

   a. Verify that $1 \cdot 2 \cdot 3 \cdot 4 = 4$ in $\mathbb{Z}_5$

   b. $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 6$ in $\mathbb{Z}_7$

   c. Evaluate $1 \cdot 2 \cdot 3$ in $\mathbb{Z}_4$

   d. Evaluate $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5$ in $\mathbb{Z}_6$

   e. Evaluate $4 \cdot 3$ in $\mathbb{Z}_4$

   f. Evaluate $4 \cdot 2$ in $\mathbb{Z}_4$

   g. Evaluate $5 \cdot 2$ in $\mathbb{Z}_5$

   h. Evaluate $5 \cdot 4$ in $\mathbb{Z}_5$

3. Make Addition Tables for each of the following:

**Remark:** Note that in the addition tables for $\mathbb{Z}_n$, the element 0 is the identity. Also, each element of the group must appear exactly once in each row and each column.

a. $\mathbb{Z}_2$

| + | 0 | 1 |
|---|---|---|
| 0 | | |
| 1 | | |

b. $\mathbb{Z}_3$

| + | 0 | 1 | 2 |
|---|---|---|---|
| 0 | | | |
| 1 | | | |
| 2 | | | |

c. $\mathbb{Z}_5$

| + | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | | | | | |
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |

d. $\mathbb{Z}_6$

| + | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | | | | | | |
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |

e. $\mathbb{Z}_7$

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | | | | | | | |
| 1 | | | | | | | |
| 2 | | | | | | | |
| 3 | | | | | | | |
| 4 | | | | | | | |
| 5 | | | | | | | |
| 6 | | | | | | | |

f. $\mathbb{Z}_8$

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | | | | | | | | |
| 1 | | | | | | | | |
| 2 | | | | | | | | |
| 3 | | | | | | | | |
| 4 | | | | | | | | |
| 5 | | | | | | | | |
| 6 | | | | | | | | |
| 7 | | | | | | | | |

4. Make multiplication tables for each of the following

**Remark:** Note that in the multiplication tables for $\mathbb{Z}_n$, the element 1 is the identity. Also, a set which contains the element 0 under the the binary operation of multiplication is **not a group.** There will be a row and a column of zeros.

a. $\mathbb{Z}_2$

| + | 0 | 1 |
|---|---|---|
| 0 | | |
| 1 | | |

b. $\mathbb{Z}_3$

| + | 0 | 1 | 2 |
|---|---|---|---|
| 0 | | | |
| 1 | | | |
| 2 | | | |

c. $\mathbb{Z}_5$

| + | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | | | | | |
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |

d. $\mathbb{Z}_6$

| + | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | | | | | | |
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |

e. $\mathbb{Z}_7$

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | | | | | | | |
| 1 | | | | | | | |
| 2 | | | | | | | |
| 3 | | | | | | | |
| 4 | | | | | | | |
| 5 | | | | | | | |
| 6 | | | | | | | |

f. $\mathbb{Z}_8$

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | | | | | | | | |
| 1 | | | | | | | | |
| 2 | | | | | | | | |
| 3 | | | | | | | | |
| 4 | | | | | | | | |
| 5 | | | | | | | | |
| 6 | | | | | | | | |
| 7 | | | | | | | | |

5. For each of the following $\mathbb{Z}_n$, list all of the elements of $\mathbb{Z}_n$ that have multiplicative inverses in $\mathbb{Z}_n$.

   **Remark:** The elements of $\mathbb{Z}_n$ that have multiplicative inverse, are exactly those **non-zero** elements of $\mathbb{Z}_n$ that are **relatively prime to** $n$.

   a. $\mathbb{Z}_6$

   b. $\mathbb{Z}_8$

   c. $\mathbb{Z}_{10}$

   d. $\mathbb{Z}_{12}$

   e. $\mathbb{Z}_{18}$

   f. $\mathbb{Z}_{20}$

6. Find all zero divisors in each of the following $\mathbb{Z}_n$

   **Remark:** The zero divisors in $\mathbb{Z}_n$ are exactly those **non-zero** elements of $\mathbb{Z}_n$ that are NOT relatively prime to $n$.

   a. $\mathbb{Z}_6$

   b. $\mathbb{Z}_8$

   c. $\mathbb{Z}_{10}$

   d. $\mathbb{Z}_{12}$

   e. $\mathbb{Z}_{18}$

   f. $\mathbb{Z}_{20}$

In exercises -, decide whether each of the given sets is a group with respect to the indicated operation. State all of the group axioms that fail to hold. If it is a group, state its order.

25. The set $\{1, 3\} \subseteq \mathbb{Z}_8$ with operation multiplication

26. The set $\{1, 3, 5\} \subseteq \mathbb{Z}_8$ with operation multiplication

27. The set $\{1, 2, 3\} \subseteq \mathbb{Z}_4$ with operation multiplication

28. The set $\{1, 2, 3, 4\} \subseteq \mathbb{Z}_5$ with operation multiplication

29. The set $\{0, 2, 4\} \subseteq \mathbb{Z}_8$ with operation multiplication

30. The set $\{0, 2, 4, 6, 8\} \subseteq \mathbb{Z}_{10}$ with operation multiplication

31. The set $\{0, 2, 4, 6, 8\} \subseteq \mathbb{Z}_{10}$ with operation addition

32. The set $\{0, 2, 4, 6\} \subseteq \mathbb{Z}_8$ with operation addition