

# MTH 4441 Subgroups and Cyclic Subgroups

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1. Let  $S(A) = \{e, \rho, \rho^2, \sigma, \gamma, \delta\}$ . Decide whether each of the following subsets is a subgroup of  $S(A)$ . If a set is NOT a subgroup, give a reason why it is not (in addition to the fact that an element may appear more than once, or not at all, in a row or column).

This is the group table for  $(S(A), \cdot)$ :

$\cdot$	$e$	$\rho$	$\rho^2$	$\sigma$	$\gamma$	$\delta$
$e$	$e$	$\rho$	$\rho^2$	$\sigma$	$\gamma$	$\delta$
$\rho$	$\rho$	$\rho^2$	$e$	$\gamma$	$\delta$	$\sigma$
$\rho^2$	$\rho^2$	$e$	$\rho$	$\delta$	$\sigma$	$\gamma$
$\sigma$	$\sigma$	$\delta$	$\gamma$	$e$	$\rho^2$	$\rho$
$\gamma$	$\gamma$	$\sigma$	$\delta$	$\rho$	$e$	$\rho^2$
$\delta$	$\delta$	$\gamma$	$\sigma$	$\rho^2$	$\rho$	$e$

- a.  $\{e, \sigma\}$

$\cdot$	$e$	$\sigma$
$e$	$e$	$\sigma$
$\sigma$	$\sigma$	$e$

- b.  $\{e, \delta\}$

$\cdot$	$e$	$\delta$
$e$	$e$	$\delta$
$\delta$	$\delta$	$e$

- c.  $\{e, \rho\}$

$\cdot$	$e$	$\rho$
$e$	$e$	$\rho$
$\rho$	$\rho$	$\rho^2$

- d.  $\{e, \rho^2\}$

$\cdot$	$e$	$\rho^2$
$e$	$e$	$\rho^2$
$\rho^2$	$\rho^2$	$\rho$

- e.  $\{e, \rho, \rho^2\}$

$\cdot$	$e$	$\rho$	$\rho^2$
$e$	$e$	$\rho$	$\rho^2$
$\rho$	$\rho$	$\rho^2$	$e$
$\rho^2$	$\rho^2$	$e$	$\rho$

f.  $\{e, \rho, \sigma\}$

$\cdot$	$e$	$\rho$	$\sigma$
$e$	$e$	$\rho$	$\sigma$
$\rho$	$\rho$	$\rho^2$	$\gamma$
$\sigma$	$\sigma$	$\delta$	$e$

g.  $\{e, \sigma, \gamma\}$

$\cdot$	$e$	$\sigma$	$\gamma$
$e$	$e$	$\sigma$	$\gamma$
$\sigma$	$\sigma$	$e$	$\rho^2$
$\gamma$	$\gamma$	$\rho$	$e$

h.  $\{e, \sigma, \gamma, \delta\}$

$\cdot$	$e$	$\sigma$	$\gamma$	$\delta$
$e$	$e$	$\sigma$	$\gamma$	$\delta$
$\sigma$	$\sigma$	$e$	$\rho^2$	$\rho$
$\gamma$	$\gamma$	$\rho$	$e$	$\rho^2$
$\delta$	$\delta$	$\rho^2$	$\rho$	$e$

2. Determine whether each of the following sets is a subgroup of  $G = \{1, -1, i, -i\}$  under multiplication. If a set is NOT a subgroup, give a reason why (in addition to the fact that an element may appear more than once, or not at all, in a row or column)..

a.  $\{1, -1\}$

$\cdot$	$1$	$-1$
$1$	$1$	$-1$
$-1$	$-1$	$1$

c.  $\{i, -i\}$

d.  $\{1, -i\}$

$\cdot$	$1$	$-i$
$1$	$1$	$-i$
$-i$	$-i$	$-1$

3. Consider the group  $(\mathbb{Z}_{16}, +)$ . List all the elements of the subgroup  $\langle 6 \rangle$ , and state its order.
4. Consider the group  $(\mathbb{Z}_{18}, +)$ . List all the elements of the subgroup  $\langle 8 \rangle$ , and state its order.
5.  $(U_{13}, \cdot)$  is a group. (Where  $U_{13} = \{1, 2, 3, \dots, 12\}$ .)
- List the elements of the subgroup  $\langle 4 \rangle$  of  $(U_{13}, \cdot)$ , and state its order.
  - List the elements of the subgroup  $\langle 8 \rangle$  of  $(U_{13}, \cdot)$ , and state its order.