

MTH 4441 Homework Exercises Set #7 - Permutations - Part 2 - Answers

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Name _____

In Exercises 1-4, decompose the given permutations into the “product” of disjoint cycles. (Remember - the “product” of the cycles is really function composition, so we proceed from right to left.)

1.

$$\sigma = (4, 6)(1, 5, 2) = (1, 5, 2)(4, 6)$$

2.

$$\phi = (1, 3, 4, 5, 6, 2)$$

3.

$$\tau = (1, 5, 8, 7)(2, 6, 3) = (2, 6, 3)(1, 5, 8, 7)$$

4.

$$\mu = (1, 2, 3, 5, 4)(7, 8) = (7, 8)(1, 2, 3, 5, 4)$$

In Exercises 5-11, decompose the given permutations into the “product” of transpositions. (Remember - the “product” of the cycles is really function composition, so we proceed from right to left.)

5.

$$\sigma = (4, 6)(1, 5)(5, 2) \quad \text{or} \quad \sigma = (1, 5)(5, 2)(4, 6)$$

6.

$$\phi = (1, 3)(3, 4)(4, 5)(5, 6)(6, 2)$$

7.

$$\tau = (1, 5)(5, 8)(8, 7)(2, 6)(6, 3) \quad \text{or} \quad \tau = (2, 6)(6, 3)(1, 5)(5, 8)(8, 7)$$

8.

$$\mu = (1, 2)(2, 3)(3, 5)(5, 4)(7, 8) \quad \text{or} \quad \mu = (7, 8)(1, 2)(2, 3)(3, 5)(5, 4)$$

9.

ω can be expressed as the “product” of transpositions in any of the following 6 ways:

$$\begin{aligned} \omega &= (1, 8) \underbrace{(3, 6)(6, 4)}_{=(3,6,4)} (5, 7) = (5, 7) \underbrace{(3, 6)(6, 4)}_{=(3,6,4)} (1, 8) = (5, 7)(1, 8) \underbrace{(3, 6)(6, 4)}_{=(3,6,4)} = (1, 8)(5, 7) \underbrace{(3, 6)(6, 4)}_{=(3,6,4)} \\ &= \underbrace{(3, 6)(6, 4)}_{=(3,6,4)} (1, 8)(5, 7) = \underbrace{(3, 6)(6, 4)}_{=(3,6,4)} (5, 7)(1, 8) \end{aligned}$$

10.

π can be expressed as the “product” of transpositions in any of the following 6 ways:

$$\begin{aligned} \pi &= \underbrace{(1, 3)(3, 4)}_{=(1,3,4)} (2, 6) \underbrace{(5, 8)(8, 7)}_{=(5,8,7)} = \underbrace{(1, 3)(3, 4)}_{=(1,3,4)} \underbrace{(5, 8)(8, 7)}_{=(5,8,7)} (2, 6) = \underbrace{(5, 8)(8, 7)}_{=(5,8,7)} \underbrace{(1, 3)(3, 4)}_{=(1,3,4)} (2, 6) \\ &= \underbrace{(5, 8)(8, 7)}_{=(5,8,7)} (2, 6) \underbrace{(1, 3)(3, 4)}_{=(1,3,4)} = (2, 6) \underbrace{(5, 8)(8, 7)}_{=(5,8,7)} \underbrace{(1, 3)(3, 4)}_{=(1,3,4)} = (2, 6) \underbrace{(1, 3)(3, 4)}_{=(1,3,4)} \underbrace{(5, 8)(8, 7)}_{=(5,8,7)} \end{aligned}$$

11.

$$\lambda = (1, 3)(3, 4)(4, 7)(7, 8)(8, 6)(6, 5)(5, 2)$$