

MTH 4441 HW #9 - Cosets

FALL 2017

Pat Rossi

Name _____

1. ~

- (a) $(4\mathbb{Z}, +)$ is a subgroup of $(\mathbb{Z}, +)$. Find all of the cosets of $4\mathbb{Z}$.
- (b) Create a group table for the Factor Group $(\mathbb{Z}/4\mathbb{Z}, +)$. (i.e., the cosets of $4\mathbb{Z}$ form a group under the operation of “coset addition.” Create a group table for the cosets of $4\mathbb{Z}$.)

2. ~

- (a) $(4\mathbb{Z}, +)$ is a subgroup of $(2\mathbb{Z}, +)$. Find all of the cosets of $4\mathbb{Z}$.
- (b) Create a group table for the Factor Group $(2\mathbb{Z}/4\mathbb{Z}, +)$. (i.e., the cosets of $4\mathbb{Z}$ form a group under the operation of “coset addition.” Create a group table for the cosets of $4\mathbb{Z}$.)

3. ~

- (a) $\langle 3 \rangle$ is a cyclic subgroup of $(\mathbb{Z}_{12}, +)$. Find all of the cosets of $\langle 3 \rangle$.
- (b) Create a group table for the Factor Group $(\mathbb{Z}_{12}/\langle 3 \rangle, +)$. (i.e., the cosets of $\langle 3 \rangle$ form a group under the operation of “coset addition.” Create a group table for the cosets of $\langle 3 \rangle$.)

4. ~

- (a) $\langle 4 \rangle$ is a cyclic subgroup of $(\mathbb{Z}_{12}, +)$. Find all of the cosets of $\langle 4 \rangle$.
- (b) Create a group table for the Factor Group $(\mathbb{Z}_{12}/\langle 4 \rangle, +)$. (i.e., the cosets of $\langle 4 \rangle$ form a group under the operation of “coset addition.” Create a group table for the cosets of $\langle 4 \rangle$.)

5. ~

- (a) Show that $\phi : (\mathbb{Z}, +) \rightarrow (\mathbb{Z}_4, +)$, given by $\phi(n) = n \bmod 4$, is a homomorphism. (The operation in $(\mathbb{Z}_4, +)$ is addition modulo 4.)
- (b) Identify $\ker(\phi)$, and compute the left and right cosets of $\ker(\phi)$.
- (c) Create a group table for the Factor Group $(\mathbb{Z}/\ker(\phi), +)$. (i.e., the cosets of $\ker(\phi)$ form a group under the operation of “coset addition.” Create a group table for the cosets of $\ker(\phi)$.)
- (d) Define an isomorphism $\mu : (\mathbb{Z}/\ker(\phi), +) \rightarrow (\mathbb{Z}_4, +)$