

MTH 4441 Test #1

FALL 2017

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Name _____

Part #1 - Definitions and Theorems

Express the definitions and statements of the theorems CLEARLY

1. Define **Group**

2. Define **isomorphism, isomorphic**

3. Define **congruent (congruence) modulo n** .

4. Define **cyclic subgroup**.

5. Name five properties of groups that are preserved by isomorphisms

6. State three theorems concerning cyclic groups

Part #2 - Proofs

7. **Prove:** The identity of a group is unique. (Do NOT appeal to the cancellation laws.)

8. **Prove:** Given a group $(G, *)$, and an element $x \in (G, *)$, the inverse of x is unique. (Do NOT appeal to the cancellation laws.)

Part #3 - Exercises

Show CLEARLY how you arrive at your answers!

9. Part of the group table for the group $(G, *) = (\{a, b, c, d\}, *)$ is given. Complete the table.

$*$	a	b	c	d
a		d		
b				
c				c
d				d

10. Find all generators of $(\mathbb{Z}_8, +)$

(a) Find all proper subgroups of $(\mathbb{Z}_8, +)$ and list their generators

(b) Draw a subgroup diagram of $(\mathbb{Z}_8, +)$

11. Construct the group table for $(\mathbb{Z}_4, +)$

$(\mathbb{Z}_4, +)$

$+$	0	1	2	3
0				
1				
2				
3				

12. Construct the group table for (U_5, \cdot)

(U_5, \cdot)

\cdot	1	2	3	4
1				
2				
3				
4				

13. With reference to exercises 11 and 12, define two different isomorphisms $\phi : (\mathbb{Z}_4, +) \rightarrow (U_5, \cdot)$

(Explain your reasoning in each case. You do NOT have to PROVE that ϕ is an isomorphism to do this.)