MTH 6610 - History of Math Reading Assignment #3 - Answers

Spring 2016

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Instructions. Read pages 72 - 102 to find the answers to these questions in your reading.

1. What manuscript reveals that the Babylonians knew the Pythagorean Theorem 1000 years before Pythagoras was born?

   The Babylonian clay tablet *Plimpton 322,* deciphered in 1945, dates between 1900 B.C. and 1600 B.C. and reveals that the Babylonians knew the so-called Pythagorean Theorem more than 1000 years before Pythagoras was born.

2. What technique did the Babylonians have for generating "Pythagorean Triples"? (i.e., numbers, a, b, c such that $a^2 + b^2 = c^2$)

   For natural numbers m, n with $m > n$, let:

   $x = 2mn$

   $y = m^2 - n^2$

   $z = m^2 + n^2$

3. What manuscript reveals that the "Late Egyptians" knew of the Pythagorean Theorem?

   The Cairo Mathematical Papyrus, unearthed in 1938, reveals that the "Late Egyptians" knew that triangles whose sides were of proportion $(3, 4, 5)$, $(5, 12, 13)$, and $(20, 21, 29)$ were right triangles.

   Furthermore, the papyrus contains 40 problems of a mathematical nature, of which 9 deal exclusively withe the Pythagorean Theorem.

4. From what time period is this manuscript from?

   The Cairo Mathematical Papyrus dates to the early Ptolemeic Dynasties (c. 300 B.C.).

5. What allowed the Greeks to progress far beyond their Babylonian and Egyptian predecessors in mathematics?

   There were a couple of factors at work here.

   First, the Greeks were dedicated to imposing their culture on "conquered territories," thus, all early Greek mathematics came - not from Greece - but from Greek outposts in Asia Minor, southern Italy, and Africa. This input that came from outside Greece also included much that had already been accomplished by the older Babylonian and Egyptian cultures.

   Second, prior to the "Golden Age of Greece," learning and knowledge had largely been monopolized by powerful priesthoods and the "educated elite" in order to preserve their power and influence. In contrast, the Greeks considered the pursuit of knowledge to be the right and duty of all citizens.

6. Name six well-known Greek mathematicians of antiquity.

   Euclid, Archimedes, Apollonius, Ptolemy, Pappus, and Diophantus.

7. What early life experiences prepared Thales for a prodigious mathematical career?

   During his early years, his commercial ventures led him to travel extensively. In his travels, he learned geometry from the Egyptians and astronomy from the Babylonians.

Remark: A recurring theme in our study of Math History is that many of the prodigious mathematicians of antiquity travelled extensively and that this travel played a significant role in their mathematical development. We might be mindful of this when some of our better students show an interest in "study abroad" programs. If the interest is there, but they are hesitant, we might offer them encouragement based on this observation.

8. What is largely considered to be Thales' greatest contribution to mathematics?

   The orderly development of theorems by rigorous proof (i.e., The use of logical proof, based on deductive reasoning - as opposed to experiment and/or intuition).

9. What "theorems" from "Euclidean Geometry" are generally credited to Thales?

   - An angle inscribed in semicircle is a right angle.
   - A circle is bisected by its diameter
   - The base angles of an isosceles triangle are equal
   - If two straight lines intersect, their opposite angles are equal.
   - The (corresponding) sides of similar triangles are proportional
   - Two triangles are congruent if they have one side and two adjacent angles, respectively, equal. (The *Angle-Side-Angle* Theorem)

10. How was Thales able to measure the height of the Great Pyramid?

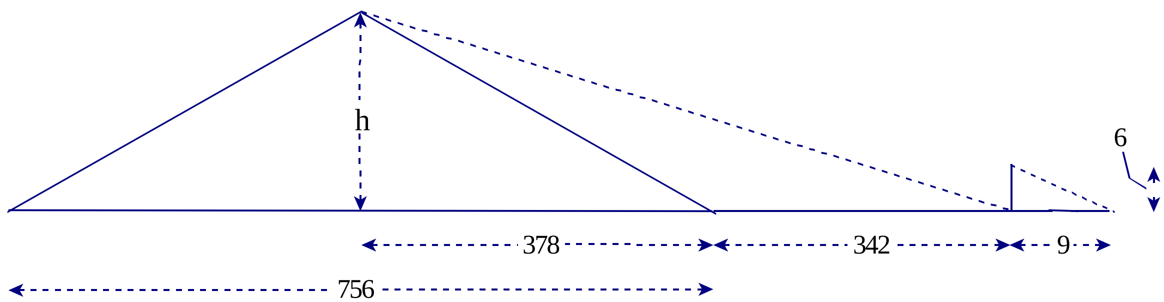
    (Referring to the picture below) Thales knew that the length of each side of the pyramid was 756 feet. Thus, the distance from the side of the base of the pyramid to the center of its base was 378 feet.

    He placed a 6 ft staff at the end of the shadow cast by the pyramid and observed that the length of the shadow was 9 ft.

    Next, he measured the distance from the staff to the side of the base of the pyramid. That distance was 342 ft.

    By similar triangles, he knew that the right triangle with legs of length h and $(378 + 342)$ was similar to the right triangle with legs of length 6 and 9.

    i.e., $\frac{h}{(378+342)} = \frac{6}{9} \Rightarrow h = \frac{(6)(378+342)}{9} = 480$ ft



11. Give two explanations as to how Thales was able to compute the distance of a ship from shore.

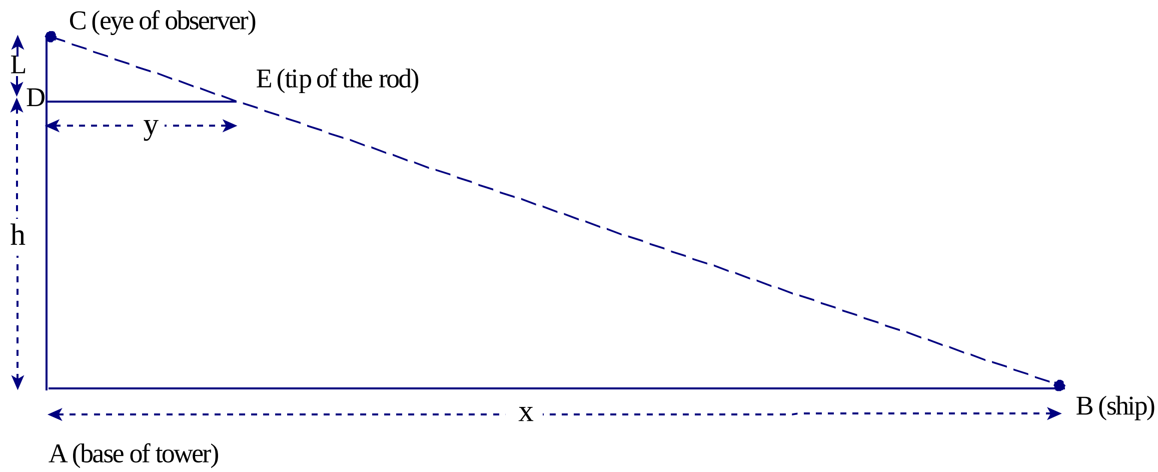
Explanation 1

(See picture below)

Thales stood atop a tower of height h. He extended a rod over the edge of the tower until the end of the rod was directly in his line of sight with the ship. He measured the length of the rod extending beyond the wall of the tower (we'll call it y.) He measured his own height (to eye level) we'll call it L. To measure the distance from the wall of the tower to the ship (we'll call that distance x), Thales used similar triangles.

(Triangle DCE is similar to triangle ACB.

Hence, by similar triangles, $\frac{x}{h+L} = \frac{y}{L} \Rightarrow x = \frac{(y)(h+L)}{L}$



Explanation 2

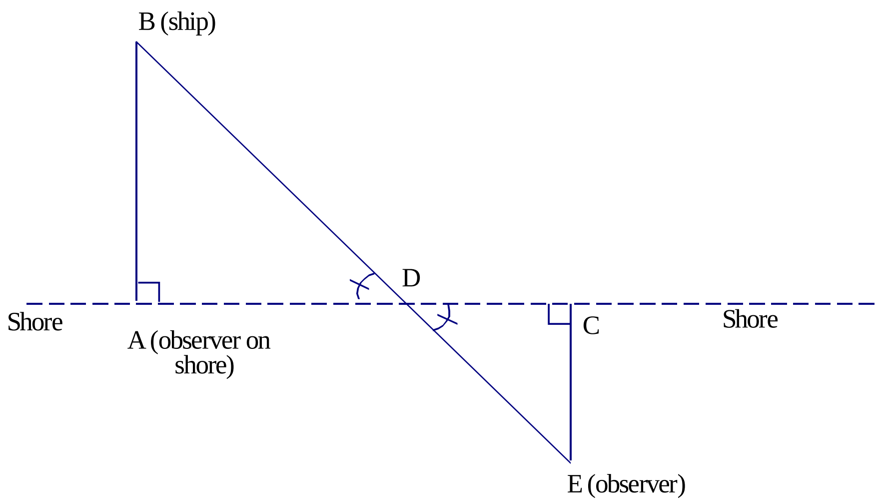
Thales stood on shore (at a point that we'll call A) so that his line of sight to the ship was perpendicular to the shore. (We'll call the location of the ship point B.)

Thales walked an arbitrary distance along the shore to a point that we'll call C. Somewhere on the shore, between points A and C, he somewhat arbitrarily picked a point (we'll call it D) that was closer to C than to A.

He then walked away from the shore (walking perpendicular to the shore) until point D was directly in his line of sight with the ship.

He measured the distances from A to D, from D to C, and from C to E. The distance from ship to shore (distance from A to B) could then be computed by similar triangles.

$\frac{|\overline{AB}|}{|\overline{AD}|} = \frac{|\overline{CE}|}{|\overline{DC}|} \Rightarrow |\overline{AB}| = \frac{|\overline{AD}||\overline{CE}|}{|\overline{DC}|}$



12. What possible link existed between Thales and Pythagoras?

    Thales may have been Pythagoras' teacher/mentor.

13. What early life experiences may have prepared Pythagoras for a prodigious mathematical career?

    He left home in Samos as a young man to study in Phoenicia and Egypt, and may have also studied in Babylonia. After years of extensive travel he settled in Crotona in southern Italy.

Remark: Note again that this person was well-travelled and that this influence no doubt played a significant role in his mathematical development. This point should not be lost on our students - *or us!* We are still developing mathematically (and in other ways) as well!

14. Where was the Pythagorean School founded?

    Crotona - a Dorian colony in southern Italy.

15. What four principal areas of study did the Pythagoreans engage in?

    Arithmetica (number theory), Harmonia (music), Geometria (geometry), and Astrologia (astronomy).

16. True or false: Women were banned from the Pythagorean School.

    False - They were allowed to attend the master's lectures. At least 28 women were in the select category of Mathematici. (This was the inner circle of the Pythagoreans, to whom all "secret" doctrines were disclosed.)

17. Who was Theano?

    Theano was Pythagoras' wife and a gifted mathematician in her own right. Her productive career evidently continued after Pythagoras' death.

18. Why is so little known, regarding the mathematical advances made by the Pythagoreans?

    Teachings in the Pythagorean school were passed along by word of mouth - not written down. Pythagoras himself apparently never put any of his results in writing. Furthermore, members of the school were bound not to disclose anything taught by the master or others in the school.

19. What particular area of study may have provided the Pythagoreans with their strongest evidence that everything in the Physical universe can be attributed to, and modeled by, numbers? (specifically, integers)

    Harmonia. It was noted (via lengths of vibrating strings) that the "most beautiful musical harmonies" (octaves, fifths, fourths, thirds, etc.) corresponded to the simplest ratios (2:1, 3:2, 4:3, etc.) with the respect to the fundamental harmonic.

Also, do the following exercises from our text:

p. 80 #1-4
p. 103 #1, 3, 6, 7