# MTH 1125 (10 am) Test #1 - Solutions

Fall 2010

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**Instructions.** Show CLEARLY how you arrive at your answers

1. Compute: $\lim_{x\to 2} \frac{x^2-x-2}{x^2+3x-10} =$

   (a) 1. Try plugging in:

   $\lim_{x\to 2} \frac{x^2-x-2}{x^2+3x-10} = \frac{(2)^2-(2)-2}{(2)^2+3(2)-10} = \frac{0}{0}$ No Good - Division by Zero

   2. Try factoring and canceling

   $\lim_{x\to 2} \frac{x^2-x-2}{x^2+3x-10} = \lim_{x\to 2} \frac{(x+1)(x-2)}{(x+5)(x-2)} = \lim_{x\to 2} \frac{(x+1)}{(x+5)} = \frac{(2)+1}{(2)+5} = \frac{3}{7}$

   i.e., $\lim_{x\to 2} \frac{x^2-x-2}{x^2+3x-10} = \frac{3}{7}$

2. Compute: $\lim_{x\to 2} \frac{2x+5}{x^2+5} =$

   (a) 1. Try plugging in:

   $\lim_{x\to 2} \frac{2x+5}{x^2+5} = \frac{2(2)+5}{(2)^2+5} = 1$

   i.e., $\lim_{x\to 2} \frac{2x+5}{x^2+5} = 1$

3. Compute: $\lim_{x\to 3} \frac{x+5}{x^2-2x-3} =$

(a) 1. Try plugging in:

$\lim_{x\to 3} \frac{x+5}{x^2-2x-3} = \frac{(3)+5}{(3)^2-2(3)-3} = \frac{8}{0}$ No Good - Division by Zero

2. Try factoring and canceling

No Good - factoring and canceling only works when Step #1 yields $\frac{0}{0}$.

3. Analyze one-sided limits

$\lim_{x\to 3^-} \frac{x+5}{x^2-2x-3} = \lim_{x\to 3^-} \frac{x+5}{(x+1)(x-3)} = \frac{8}{(4)(-\varepsilon)} = \frac{2}{(-\varepsilon)} = -\infty$

$$\begin{aligned} & x \to 3^- \\ \Rightarrow\quad & x < 3 \\ \Rightarrow\quad & x - 3 < 0 \end{aligned}$$

$\lim_{x\to 3^+} \frac{x+5}{x^2-2x-3} = \lim_{x\to 3^+} \frac{x+5}{(x+1)(x-3)} = \frac{8}{(4)(+\varepsilon)} = \frac{2}{(\varepsilon)} = +\infty$

$$\begin{aligned} & x \to 3^+ \\ \Rightarrow\quad & x > 3 \\ \Rightarrow\quad & x - 3 > 0 \end{aligned}$$

Since the one-sided limits are not equal, $\lim_{x\to 3} \frac{x+5}{x^2-2x-3}$ Does Not Exist.

4. Compute: $\lim_{x\to 2} \frac{\sqrt{x+7}-3}{x-2} =$

(a) 1. Try plugging in:

$\lim_{x\to 2} \frac{\sqrt{x+7}-3}{x-2} = \frac{\sqrt{(2)+7}-3}{(2)-2} = 0$ No Good - Division by Zero

2. Try factoring and canceling

$\lim_{x\to 2} \frac{\sqrt{x+7}-3}{x-2} = \lim_{x\to 2} \frac{\sqrt{x+7}-3}{x-2} \cdot \frac{\sqrt{x+7}+3}{\sqrt{x+7}+3} = \lim_{x\to 2} \frac{\left(\sqrt{x+7}\right)^2-(3)^2}{(x-2)\left[\sqrt{x+7}+3\right]} =$

$\lim_{x\to 2} \frac{(x+7)-9}{(x-2)\left[\sqrt{x+7}+3\right]} = \lim_{x\to 2} \frac{x-2}{(x-2)\left[\sqrt{x+7}+3\right]} = \lim_{x\to 2} \frac{1}{\left[\sqrt{x+7}+3\right]} =$

$\frac{1}{\left[\sqrt{(2)+7}+3\right]} = \frac{1}{3+3} = \frac{1}{6}$

i.e., $\lim_{x\to 2} \frac{\sqrt{x+7}-3}{x-2} = \frac{1}{6}$

5. Compute: $\lim_{x\to\infty} \frac{5x^3+3x+2}{5x^3+5x^2+5} =$

$\lim_{x\to\infty} \frac{5x^3+3x+2}{5x^3+5x^2+5} = \lim_{x\to\infty} \frac{5x^3}{5x^3} = \lim_{x\to\infty} 1 = 1$

i.e., $\lim_{x\to\infty} \frac{5x^3+3x+2}{5x^3+5x^2+5} = 1$

6.

| $x =$ | $f(x) =$ |
|---|---|
| | |
| 1.5 | 15.1 |
| 1.9 | 227.8 |
| 1.99 | 1212.3 |
| 1.999 | 21156.3 |
| 1.9999 | 834561.9 |

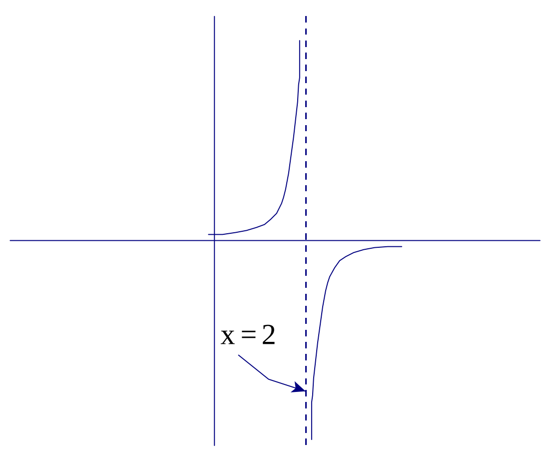
| $x =$ | $f(x) =$ |
|---|---|
| | |
| 2.5 | $-15.1$ |
| 2.1 | $-227.8$ |
| 2.01 | $-1212.3$ |
| 2.001 | $-21156.3$ |
| 2.0001 | $-834561.9$ |

Based on the information in the table above, do the following:

(a) $\lim_{x\to 2^-} f(x) = +\infty$

(b) $\lim_{x\to 2^+} f(x) = -\infty$

(c) Graph $f(x)$



7. Determine whether $f(x)$ is continuous at the point $x = 3$.

$$f(x) = \begin{cases} \frac{x^2-2x-3}{x-3} & \text{for } x < 3 \\ \\ 2x - 3 & \text{for } x \geq 3 \end{cases}$$

$f(x)$ is continuous at the point $x = 3$ exactly when $\lim_{x\to 3} f(x) = f(3)$.

We will check this by computing $\lim_{x\to 3} f(x)$.

Since $f(x)$ is defined differently for $x < 3$ than it is for $x > 3$, we must compute the one-sided limits in order to compute $\lim_{x\to 3} f(x)$.

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-2x-3}{x-3} = \lim_{x\to 3^-} \frac{(x+1)(x-3)}{(x-3)} = \lim_{x\to 3^-} (x+1) = ((3)+1) = 4$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (2x - 3) = (2(3) - 3) = 3$

Since the one-sided limits are not equal, $\lim_{x\to 3} f(x)$ does not exist.

Since $\lim_{x\to 3} f(x)$ does not exist, $\lim_{x\to 3} f(x) \neq f(3)$.

| |
|---|
| Since $\lim_{x\to 3} f(x) \neq f(3)$, $f(x)$ is NOT continuous at the point, $x = 3$. |

8. $f(x) = \frac{x^3}{x^2-4}$. Find the asymptotes and graph

| Verticals |
|---|

Find $x$-values that cause division by zero.

$\Rightarrow x^2 - 4 = 0$

$\Rightarrow (x+2)(x-2) = 0$

$\Rightarrow x = -2$ and $x = 2$ *possible* vertical asymptotes

Compute the one-sided limits

$x = -2$

$\lim_{x\to -2^-} \frac{x^3}{x^2-4} = \lim_{x\to -2^-} \frac{x^3}{(x+2)(x-2)} = \frac{-8}{(-\varepsilon)(-4)} = \frac{-8}{(\varepsilon)(4)} = \frac{(-2)}{\varepsilon} = -\infty$

$$\begin{array}{ll} & x \to -2^- \\ \Rightarrow & x < -2 \\ \Rightarrow & x + 2 < 0 \end{array}$$

Infinite limits tell us that $x = -2$ IS a vertical asymptote

$\lim_{x\to -2^+} \frac{x^3}{x^2-4} = \lim_{x\to -2^+} \frac{x^3}{(x+2)(x-2)} = \frac{-8}{(\varepsilon)(-4)} = \frac{8}{(\varepsilon)(4)} = \frac{(2)}{\varepsilon} = +\infty$

$$\begin{array}{ll} & x \to -2^+ \\ \Rightarrow & x > -2 \\ \Rightarrow & x + 2 > 0 \end{array}$$

$x = 2$

$\lim_{x\to 2^-} \frac{x^3}{x^2-4} = \lim_{x\to 2^-} \frac{x^3}{(x+2)(x-2)} = \frac{8}{(4)(-\varepsilon)} = \frac{(-2)}{\varepsilon} = -\infty$

$$\begin{array}{ll} & x \to 2^- \\ \Rightarrow & x < 2 \\ \Rightarrow & x - 2 < 0 \end{array}$$

Infinite limits tell us that $x = 2$ IS a vertical asymptote

$\lim_{x\to 2^+} \frac{x^3}{x^2-4} = \lim_{x\to 2^+} \frac{x^3}{(x+2)(x-2)} = \frac{8}{(4)(\varepsilon)} = \frac{(2)}{\varepsilon} = +\infty$

$$\begin{array}{ll} & x \to 2^+ \\ \Rightarrow & x > 2 \\ \Rightarrow & x - 2 > 0 \end{array}$$

Horizontals

Compute limits as $x \to -\infty$ and $x \to +\infty$.

$\lim_{x\to-\infty} \frac{x^3}{x^2-4} = \lim_{x\to-\infty} \frac{x^3}{x^2} = \lim_{x\to-\infty} x = -\infty$

Limits are not *finite* and *constant.* Hence, there is NO horizontal asymptote

$\lim_{x\to+\infty} \frac{x^3}{x^2-4} = \lim_{x\to+\infty} \frac{x^3}{x^2} = \lim_{x\to+\infty} x = +\infty$

Graph $f(x) = \frac{x^3}{x^2-4}$