# MTH 6610 - History of Math Reading Assignment #2 Answers

Term V - 2024

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**Instructions.** Read pages 33-71 to find the answers to these questions in your reading.

1. According to Aristotle, where did mathematics begin, and why?

   The Mathematical Sciences originated in Egypt, because here the priestly class was allowed leisure. (History confirms this opinion - The most spectacular advances in math have occurred contemporaneously with a leisure class devoted to the pursuit of knowledge.)

2. From what manuscripts do we derive most of our knowledge of ancient Egyptian mathematics?

   (From two manuscripts - The Rhind Papyrus and the Golenischev Papyrus (a.k.a. the Moscow Papyrus)

3. What is the significance of the Rosetta Stone to the history of mathematics?

   The Rhind Papyrus, written in hieratic script (a cursive form of hieroglyphics) gave a fairly exhaustive characterization of Ancient Egyptian Mathematics. However, an understanding of how to read hieroglyphics and demotic (hieratic) script had been lost for centuries. The Rosetta Stone, which contained hieroglyphic, demotic, and Greek versions of the same text (a decree of Ptolemy V), provided the means for linguistic scholars to decipher hieroglyphics and demotic (hieratic) script. Thus, the contents of the Rhind Papyrus were made accessible to the world of Mathematics.

4. Briefly describe how the Egyptians performed multiplication and division.

   Egyptian multiplication is based on the fact that every positive integer can be expressed as the sum of distinct powers of 2.

   Hence, Egyptians performed multiplication by:

   (a) Expressing the multiplier as the sum of distinct powers of 2

   (b) Multiplying the multiplicand by each of these powers of 2

   (c) Computing the sum of these products

Analogously, division was performed by:

(a) Multiplying the divisor by successively larger powers of 2

(b) Stopping the process when a number larger than the dividend was obtained

(c) Adding up exactly those multiples of the divisor that were required to yield the dividend and eliminating the others.

(d) Adding up those multiples of 2 that corresponded to the multiples of the divisor that were required to yield dividend.

5. What was peculiar about the Egyptians' use and notation of fractions?

   With the exception of the fraction $\frac{2}{3}$, the Egyptians used only unit fractions (fractions of the form $\frac{1}{n}$).

6. To what extent were the Egyptians capable of solving problems that can be modeled by linear equations, and what do we call the methods that they used?

   Egyptians could solve problems that could be modeled by linear equations of the form

$$x + \frac{b}{a}x = c,$$

   using the Method of False Position and

$$ax + b = 0.$$

   using the Method of Double False Position.

7. How did the Egyptians compute the area of a four-sided piece of land? What is noteworthy about this procedure?

   The area was calculated by computing the product of the averages of the pairs of the lengths of the opposite sides. i.e., given a four-sided figure with consecutive sides of length a, b, c, d, the area was calculated to be

$$A = \left[\frac{1}{2}(a+c)\right]\left[\frac{1}{2}(b+d)\right]$$

   or

$$A = \frac{1}{4}(a+c)(b+d).$$

   It is noteworthy that the formula is incorrect and only comes close to being accurate when the field (figure) is approximately rectangular.

   It is also noteworthy that 3000 years earlier, the Babylonians used the same incorrect formula.

8. What formula did the Egyptians use to compute the area of a circle?

   Given a circle with diameter $d$, the area was presumed to be

   $$A = \left(d - \frac{d}{9}\right)^2$$

   or

   $$A = \left(\frac{8d}{9}\right)^2$$

   This is equivalent to $A = \pi r^2$, using $\pi \approx \frac{256}{81} = 3.1605$.

9. What noteworthy theorem from mathematics is attributable to the "Old Babylonians" (1800-1600 B.C.)?

   There are two answers to this one. First, they knew the so-called Pythagorean Theorem. Second, they had a formula for the circumference of a circle

   $$C = 3d$$

   where $C$ is the circumference, $d$ is the diameter,and 3 is playing the role of $\pi$.

10. What explanation is given as to why the Babylonians attained a much more sophisticated and theoretical understanding of mathematics?

    This is attributed to their "remarkably facile" number system. Their sexagesimal notation enabled them to calculate with fractions as readily as with integers and led to a highly developed algebra.

**Remark:** We should take note of the fact that something as seemingly arbitrary as notation would turn out to be so indispensable in enabling the Babylonians to make giant strides in mathematics and that a poor system of notation almost certainly prevented the Egyptians from doing the same. I believe that there is a message here for our students, many of whom typically think that how they write their work and document their work is irrelevant and should be left strictly up to them. This historical example may come in handy in convincing our more submissive students that there are sound reasons why they should do things "our way" or "the conventional way," instead of doing things "their way." Armed with this fact, we may want to encourage our students not to let lack of good notation prevent *them* from making giant strides in mathematics!

**Remark:** More examples of how the "right" notation proved indispensable in enabling mathematicians to make giant strides and how lack of the "right" notation prevented other mathematicians from doing the same will appear in our reading.

11. What "algebraic" concept of division did the Babylonians have?

    They interpreted "$a$ divided by $b$" to mean "$a$ multiplied by the *reciprocal* of $b$." (i.e., $a \div b$ is the same as $a \cdot \frac{1}{b}$). This is essentially the same as viewing the fraction $\frac{1}{b}$ as the *multiplicative inverse* of $b$.

12. What note-worthy "algebraic" tool had the Babylonians developed by 2000 B.C.?

The equivalent of the quadratic formula for solving quadratic equations of the form

$$x^2 + Ax = B.$$

In terms of "the standard form of the quadratic equation"

$$ax^2 + bx + c = 0,$$

$a = 1$, $b = A$, and $c = -B$.

Their verbal instructions for solving (word problems that are modeled by) a quadratic equation are equivalent to the formula

$$x = \sqrt{\left(\frac{A}{2}\right)^2 + B} - \frac{A}{2}$$

Which is mathematically equivalent to

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ where } a = 1.$$

**Remark:** This "formula" only yields one root. Otherwise, this formula is equivalent to "our" quadratic formula, since the quadratic equation

$$ax^2 + bx + c = 0$$

can be reduced to the form

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

by dividing both sides by $a$ and subtracting $\frac{c}{a}$ from both sides.

**Homework:**

**On page 51,** do the following exercises: 1-3, 11, 12, 19, 20

Solve the equations below using double false position. (you need not express the results using unit fractions)

$3x + 16 = 0$

$16x + 3 = 0$

**Remark:** Note that without the concept of "equation," the solution of problems that can be modeled by equations of the form:

$$x + \frac{b}{a}x = c,$$

and

$$ax + b = 0.$$

have no obvious method of solution. Hence, the Egyptians had the "mysterious algorithms" used in the Method of False Position and the Method of Double False Position.

**On page 61,** do the following exercises: 1a, 2, 3, 4