MTH 1125 2pm Class - Test #4 - Solutions

Fall 2020

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Show CLEARLY how you arrive at your answers!

1. **Compute:** $\int \left(20x^3 + 36x^2 - 12x + 9\sqrt{x} + 4\right) dx =$

$$\int \left(20x^3 + 36x^2 - 12x + 9\sqrt{x} + 4\right) dx = \int \left(20x^3 + 36x^2 - 12x + 9x^{\frac{1}{2}} + 4\right) dx$$

$$= 20\left[\frac{x^4}{4}\right] + 36\left[\frac{x^3}{3}\right] - 12\left[\frac{x^2}{2}\right] + 9\left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right] + 4x + C = 5x^4 + 12x^3 - 6x^2 + 6x^{\frac{3}{2}} + 4x + C$$

$$\boxed{\text{i.e., } \int \left(20x^3 + 36x^2 - 12x + 9\sqrt{x} + 4\right) dx = 5x^4 + 12x^3 - 6x^2 + 6x^{\frac{3}{2}} + 4x + C}$$

2. **Compute:** $\int \left(6x^2+24x+8\right)^4 (x+2)\,dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\left(6x^2+24x+8\right)^4$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(6x^2+24x+8\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(6x^2+24x+8\right)}_{\text{function}} - - - - \rightarrow \underbrace{(x+2)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(6x^2+24x+8\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 6x^2+24x+8 \\ \Rightarrow \tfrac{du}{dx} &= 12x+24 \\ \Rightarrow du &= (12x+24)\,dx \\ \Rightarrow \tfrac{1}{12}du &= (x+2)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\int \underbrace{\left(6x^2+24x+8\right)^4}_{u^4}\underbrace{(x+2)\,dx}_{\frac{1}{12}du} = \int u^4 \tfrac{1}{12}du = \tfrac{1}{12}\int u^4 du$$

4. Integrate (in terms of u).

$$\tfrac{1}{12}\int u^4 du = \tfrac{1}{12}\left[\tfrac{u^5}{5}\right] + C = \tfrac{1}{60}u^5 + C$$

5. Re-express in terms of the original variable, x.

$$\int \left(6x^2+24x+8\right)^4 (x+2)\,dx = \underbrace{\frac{1}{60}\left(6x^2+24x+8\right)^5 + C}_{\frac{1}{60}u^5+C}$$

$$\text{i.e., } \int \left(6x^2+24x+8\right)^4 (x+2)\,dx = \tfrac{1}{60}\left(6x^2+24x+8\right)^5 + C$$

3. **Compute:** $\int \big(2\cos(x) - 5\csc^2(x) + 2\sec(x)\tan(x)\big)\, dx =$

$\int \big(2\cos(x) - 5\csc^2(x) + 2\sec(x)\tan(x)\big)\, dx = 2[\sin(x)] - 5[-\cot(x)] + 2[\sec(x)] + C$

$= 2\sin(x) + 5\cot(x) + 2\sec(x) + C$

$$\boxed{\text{i.e., } \int \big(2\cos(x) - 5\csc^2(x) + 2\sec(x)\tan(x)\big)\, dx = 2\sin(x) + 5\cot(x) + 2\sec(x) + C}$$

4. **Compute:** $\int \sin\left(5x^3+6x+3\right)\left(5x^2+2\right)dx =$

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\quad \sin\left(5x^3+6x+3\right)$

   $\nearrow \qquad \nwarrow$

   outer $\qquad\qquad$ inner

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = 5x^3+6x+3$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\quad \underbrace{5x^3+6x+3}_{\text{function}} - - - - \rightarrow \underbrace{\left(5x^2+2\right)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = 5x^3+6x+3$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$
\begin{aligned}
u &= 5x^3+6x+3 \\
\Rightarrow \frac{du}{dx} &= 15x^2+6 \\
\Rightarrow du &= \left(15x^2+6\right)dx \\
\Rightarrow \tfrac{1}{3}du &= \left(5x^2+2\right)dx
\end{aligned}
$$

3. Analyze in terms of u and du

$$\int \underbrace{\sin\left(5x^3+6x+3\right)}_{\sin(u)}\underbrace{\left(5x^2+2\right)dx}_{\frac{1}{3}du} = \int \sin(u)\ \tfrac{1}{3}du = \tfrac{1}{3}\int \sin(u)\,du$$

4. Integrate (in terms of u).

$$\tfrac{1}{3}\int \sin(u)\,du = \tfrac{1}{3}\left[-\cos(u)\right] + C = -\tfrac{1}{3}\cos(u) + C$$

5. Re-express in terms of the original variable x.

$$\int \sin\left(5x^3+6x+3\right)\left(5x^2+2\right)dx = \underbrace{-\frac{1}{3}\cos\left(5x^3+6x+3\right)+C}_{-\frac{1}{3}\cos(u)+C}$$

i.e., $\int \sin\left(5x^3+6x+3\right)\left(5x^2+2\right)dx = -\frac{1}{3}\cos\left(5x^3+6x+3\right)+C$

5. **Compute:** $\int_{-1}^{1}\left(6x^3+6x^2+2\right)dx =$

$$\int_{-1}^{1}\underbrace{\left(6x^3+6x^2+2\right)}_{f(x)}dx = \underbrace{\left[6\frac{x^4}{4}+6\frac{x^3}{3}+2x\right]_{-1}^{1}}_{F(x)} = \underbrace{\left[\frac{3}{2}x^4+2x^3+2x\right]_{-1}^{1}}_{F(x)} =$$

$$= \underbrace{\left[\frac{3}{2}(1)^4+2(1)^3+2(1)\right]}_{F(1)} - \underbrace{\left[\frac{3}{2}(-1)^4+2(-1)^3+2(-1)\right]}_{F(-1)} = \tfrac{11}{2}-\left(-\tfrac{5}{2}\right) = 8$$

i.e., $\int_{-1}^{1}\left(6x^3+6x^2+2\right)dx = 8$

6. **Compute:** $\int_0^1 \left(x^2+1\right)^4 x\ dx =$ (The answer may not be a whole number)

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\left(x^2+1\right)^4$ (A function raised to a power is *always* a composite function!)

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = \left(x^2+1\right)$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(x^2+1\right)}_{\text{function}} - - - - - \rightarrow \underbrace{x}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = \left(x^2+1\right)$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= x^2+1 \\ \Rightarrow \frac{du}{dx} &= 2x \\ \Rightarrow du &= 2x\,dx \\ \Rightarrow \frac{1}{2}du &= x\ dx \end{aligned}$$

When $x = 0$, $u = x^2+1 = (0)^2+1 = 1$
When $x = 1$, $u = x^2+1 = (1)^2+1 = 2$

3. Analyze in terms of u and du

$$\int_{x=1}^{x=2} \underbrace{\left(x^2+1\right)^4}_{u^4} \underbrace{x\ dx}_{\frac{1}{2}du} = \int_{u=1}^{u=2} u^4 \cdot \frac{1}{2}du = \frac{1}{2}\int_{u=1}^{u=2} u^4 du$$

Don't forget to re-write the limits of integration in terms of u!

4. Integrate (in terms of u).

$$\frac{1}{2}\int_{u=1}^{u=2} u^4 du = \frac{1}{2}\left[\frac{u^5}{5}\right]_{u=1}^{u=2} = \left[\frac{u^5}{10}\right]_{u=1}^{u=2} = \underbrace{\frac{(2)^5}{10}}_{F(2)} - \underbrace{\frac{(1)^5}{10}}_{F(1)} = \frac{32}{10} - \left(\frac{1}{10}\right) = \frac{31}{10}$$

i.e., $\int_{x=0}^{x=1} \left(x^2+1\right)^4 x\ dx = \frac{31}{10}$

7. **Compute:** $\frac{d}{dx}\left[\ln\left(5x^3+4x^2-2x\right)\right] =$

$$\underbrace{\frac{d}{dx}\left[\ln\left(5x^3+4x^2-2x\right)\right]}_{\frac{d}{dx}[\ln(g(x))]} = \underbrace{\frac{1}{5x^3+4x^2-2x}}_{\frac{1}{g(x)}} \cdot \underbrace{\left(15x^2+8x-2\right)}_{g'(x)} = \frac{15x^2+8x-2}{5x^3+4x^2-2x}$$

i.e., $\frac{d}{dx}\left[\ln\left(5x^3+4x^2-2x\right)\right] = \frac{15x^2+8x-2}{5x^3+4x^2-2x}$

8. **Compute:** $\int \frac{3x+2}{3x^2+4x+5}dx =$

$$\int \frac{3x+2}{3x^2+4x+5}dx \underbrace{=}_{\text{re-write}} \int \frac{1}{3x^2+4x+5}\ (3x+2)\ dx$$

Remark: Note that we have an approximate function/derivative pair, with the "function" in the denominator. This usually indicates that u-substitution is appropriate, with the result being a natural logarithm.

1. Is u-sub appropriate?

   a. Is there a composite function?

   Yes! $\frac{1}{3x^2+4x+5}$ is the same as $\left(3x^2+4x+5\right)^{-1}$, so it is a function raised to a power.

   Let $u =$ the "inner" of the composite function

   $\Rightarrow u = 3x^2 + 4x + 5$

   b. Is there an (approximate) function/derivative pair?

   Yes! $\underbrace{\left(3x^2+4x+5\right)}_{\text{function}} - - - - \rightarrow \underbrace{(3x+2)}_{\text{deriv}}$

   Let $u =$ the "function" of the function/deriv pair

   $\Rightarrow u = 3x^2 + 4x + 5$

   c. Is the "function" of the function/deriv pair the same as the "inner" of the composite function?

   (i.e., do criteria **a** and **b** suggest the same choice of u ?)

   Yes! $\Rightarrow$ u-substitution is appropriate

2. Compute du

$$\begin{aligned} u &= 3x^2+4x+5 \\ \Rightarrow \frac{du}{dx} &= 6x+4 \\ \Rightarrow du &= (6x+4)\,dx \\ \Rightarrow \frac{1}{2}du &= (3x+2)\,dx \end{aligned}$$

3. Analyze in terms of u and du

$$\underbrace{\int \frac{1}{3x^2+3x+5}}_{\frac{1}{u}} \underbrace{(3x+2)\ dx}_{\frac{1}{2}du} = \int \frac{1}{u}\frac{1}{2}du = \frac{1}{2}\int \frac{1}{u}du$$

4. Integrate (in terms of u).

   $\frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln|u| + C$

5. Re-express in terms of the original variable, x.

$$\int \frac{3x+2}{3x^2+4x+5}dx = \underbrace{\frac{1}{2}\ln\left|3x^2+3x+5\right| + C}_{\frac{1}{2}\ln|u|+C}$$

i.e., $\int \frac{3x+2}{3x^2+4x+5}dx = \frac{1}{2}\ln\left|3x^2+3x+5\right| + C$