MTH 1125 Test #1 - Solutions

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 3} \frac{2x^2+3x-7}{x^2+1} =$

Step #1 Try Plugging In:

$$\lim_{x\to 3} \frac{2x^2+3x-7}{x^2+1} = \frac{2(3)^2+3(3)-7}{(3)^2+1} = \frac{20}{10} = 2$$

i.e., $\lim_{x\to 3} \frac{2x^2+3x-7}{x^2+1} = 2$

2. Compute: $\lim_{x\to 1} \frac{x^2-4x+3}{x^2-3x+2} =$

$$\lim_{x\to 1} \frac{x^2-4x+3}{x^2-3x+2} = \frac{(1)^2-4(1)+3}{(1)^2-3(1)+2} = \frac{0}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Canceling:

$$\lim_{x\to 1} \frac{x^2-4x+3}{x^2-3x+2} = \lim_{x\to 1} \frac{(x-3)(x-1)}{(x-2)(x-1)} = \lim_{x\to 1} \frac{(x-3)}{(x-2)} = \frac{(1)-3}{(1)-2} = \frac{-2}{-1} = 2$$

i.e., $\lim_{x\to 1} \frac{x^2-4x+3}{x^2-3x+2} = 2$

3. $\lim_{x\to\ -3} \frac{x^2+4x-9}{x^2+x-6}$

Step #1 Try Plugging in:

$$\lim_{x\to\ -3} \frac{x^2+4x-9}{x^2+x-6} = \frac{(-3)^2+4(-3)-9}{(-3)^2+(-3)-6} = \frac{-12}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Canceling:

No Good! "Factoring and Canceling" only works when Step #1 yields $\frac{0}{0}$.

Step #3 Analyze the one-sided limits:

$$\lim_{x\to\ -3^-} \frac{x^2+4x-9}{x^2+x-6} = \lim_{x\to\ -3^-} \frac{x^2+4x-9}{(x+3)(x-2)} = \frac{-12}{(-\varepsilon)(-5)} = \frac{\left(\frac{-12}{-5}\right)}{(-\varepsilon)} = \frac{\left(\frac{12}{5}\right)}{(-\varepsilon)} = -\infty$$

$$\begin{aligned} & x \to -3^- \\ \Rightarrow\ & x < -3 \\ \Rightarrow\ & x+3 < 0 \end{aligned}$$

$$\lim_{x\to\ -3^+} \frac{x^2+4x-9}{x^2+x-6} = \lim_{x\to\ -3^+} \frac{x^2+4x-9}{(x+3)(x-2)} = \frac{-12}{(\varepsilon)(-5)} = \frac{\left(\frac{-12}{-5}\right)}{(\varepsilon)} = \frac{\left(\frac{12}{5}\right)}{(\varepsilon)} = +\infty$$

$$\begin{aligned} & x \to -3^+ \\ \Rightarrow\ & x > -3 \\ \Rightarrow\ & x+3 > 0 \end{aligned}$$

Since the one-sided limits are not equal, $\lim_{x\to\ -3} \frac{x^2+4x-9}{x^2+x-6}$ **Does Not Exist!**

4. Compute: $\lim_{x\to\ -\infty} \frac{6x^4-3x^3+3}{x^4-5x+4} =$

$$\lim_{x\to\ -\infty} \frac{6x^4-3x^3+3}{x^4-5x+4} = \lim_{x\to\ -\infty} \frac{6x^4}{x^4} = \lim_{x\to\ -\infty} \frac{6}{1} == \lim_{x\to\ -\infty} 6 = 6$$

i.e., $\lim_{x\to\ -\infty} \frac{6x^4-3x^3+3}{x^4-5x+4} = 6$

5. Find the asymptotes and graph: $f(x) = \frac{x^2-5x+4}{x^2-9}$

Verticals

1. Find x-values that cause division by zero.

$\Rightarrow x^2 - 9 = 0$

$\Rightarrow (x+3)(x-3) = 0$

$\Rightarrow x = -3$ and $x = 3$ are possible vertical asymptotes.

2. Compute the one-sided limits.

$$\lim_{x\to\ -3^-} \frac{x^2-5x+4}{x^2-9} = \lim_{x\to\ -3^-} \frac{x^2-5x+4}{(x+3)(x-3)} = \frac{28}{(-\varepsilon)(-6)} = \frac{28}{(\varepsilon)(6)} = \frac{\left(\frac{28}{6}\right)}{\varepsilon} = +\infty$$

$$\begin{array}{|ll|}\hline & x \to -3^- \\ \Rightarrow & x < -3 \\ \Rightarrow & x+3 < 0 \\ \hline\end{array}$$

$$\lim_{x\to\ -3^+} \frac{x^2-5x+4}{x^2-9} = \lim_{x\to\ -3^+} \frac{x^2-5x+4}{(x+3)(x-3)} = \frac{28}{(+\varepsilon)(-6)} = \frac{\left(\frac{28}{-6}\right)}{\varepsilon} = \frac{\left(-\frac{28}{6}\right)}{\varepsilon} = -\infty$$

$$\begin{array}{|ll|}\hline & x \to -3^+ \\ \Rightarrow & x > -3 \\ \Rightarrow & x+3 > 0 \\ \hline\end{array}$$

Since the one-sided limits are infinite, $x = -3$ *is a vertical asymptote.*

$$\lim_{x\to\ 3^-} \frac{x^2-5x+4}{x^2-9} = \lim_{x\to\ 3^-} \frac{x^2-5x+4}{(x+3)(x-3)} = \frac{-2}{(6)(-\varepsilon)} = \frac{\left(\frac{-2}{6}\right)}{(-\varepsilon)} = \frac{\left(-\frac{2}{6}\right)}{(-\varepsilon)} = \infty$$

$$\begin{array}{|ll|}\hline & x \to 3^- \\ \Rightarrow & x < 3 \\ \Rightarrow & x-3 < 0 \\ \hline\end{array}$$

$$\lim_{x\to 3^+} \frac{x^2-5x+4}{x^2-9} = \lim_{x\to 3^+} \frac{x^2-5x+4}{(x+3)(x-2)} = \frac{-2}{(6)(+\varepsilon)} = \frac{\left(\frac{-2}{6}\right)}{(+\varepsilon)} = \frac{\left(-\frac{2}{6}\right)}{(+\varepsilon)} = -\infty$$

$$\begin{array}{|ll|}\hline & x \to 3^+ \\ \Rightarrow & x > 3 \\ \Rightarrow & x-3 > 0 \\ \hline\end{array}$$

Since the one-sided limits are **infinite,** $x = 3$ *is a vertical asymptote.*

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to\ -\infty} \frac{x^2-5x+4}{x^2-9} = \lim_{x\to\ -\infty} \frac{x^2}{x^2} = \lim_{x\to\ -\infty} 1 = 1$

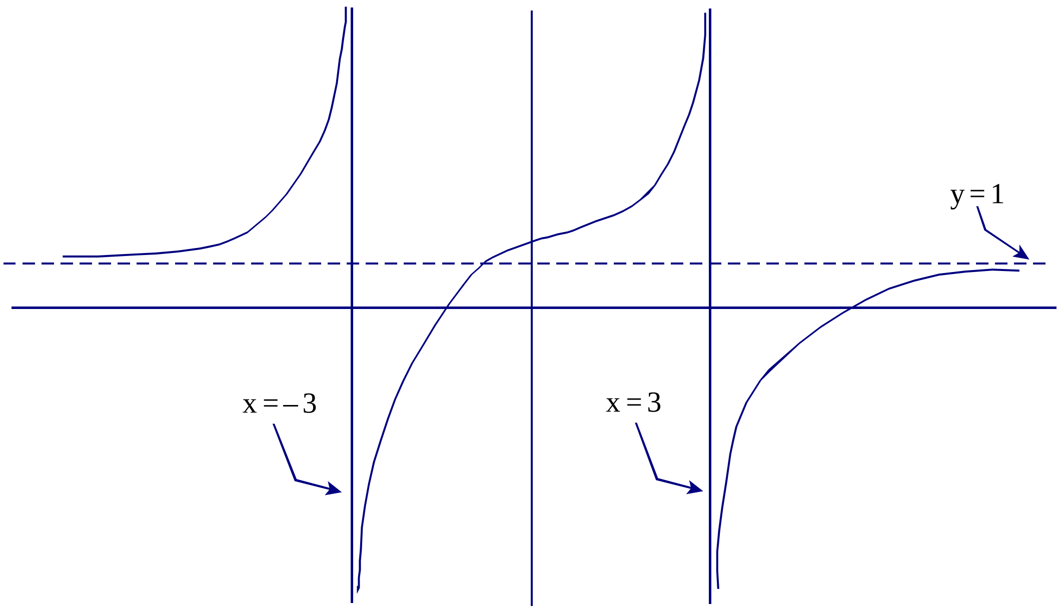
$\lim_{x\to+\infty} \frac{x^2-5x+4}{x^2-9} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$

Since the limits are **finite** and **constant,** $y = 1$ *is a horizontal asymptote.*

Summary:

$\lim_{x\to-3^-} \frac{x^2-5x+4}{x^2-9} = +\infty$	
$\lim_{x\to-3^+} \frac{x^2-5x+4}{x^2-9} = -\infty$	$\lim_{x\to\ -\infty} \frac{x^2-5x+4}{x^2-9} = 1$
$\lim_{x\to3^-} \frac{x^2-5x+4}{x^2-9} = -\infty$	$\lim_{x\to+\infty} \frac{x^2-5x+4}{x^2-9} = 1$
$\lim_{x\to3^+} \frac{x^2-5x+4}{x^2-9} = +\infty$	

Graph $f(x) = \frac{x^2-5x+4}{x^2-9}$



6. Compute: $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} =$

Step #1 Try Plugging in:

$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \frac{\sqrt{(3)+1}-2}{(3)-3} = \frac{0}{0}$ No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} \cdot \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \lim_{x\to 3} \frac{\left(\sqrt{x+1}\right)^2-(2)^2}{(x-3)\left[\sqrt{x+1}+2\right]}$$

$$= \lim_{x\to 3} \frac{(x+1)-4}{(x-3)\left[\sqrt{x+1}+2\right]} = \lim_{x\to 3} \frac{(x-3)}{(x-3)\left[\sqrt{x+1}+2\right]} = \lim_{x\to 3} \frac{1}{\left[\sqrt{x+1}+2\right]}$$

$$= \frac{1}{\left[\sqrt{(3)+1}+2\right]} = \frac{1}{[2+2]} = \frac{1}{4}$$

i.e., $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \frac{1}{4}$

7.

$x =$	$f(x) =$
-3.5	-10.2
-3.1	-157.32
-3.01	-10045.56
-3.001	$-235,402.27$
-3.0001	$-5,873,002.16$

$x =$	$f(x) =$
-2.5	10.2
-2.9	157.32
-2.99	10045.56
-2.999	$235,402.27$
-2.9999	$5,873,002.16$

Based on the information in the table above, do the following:

(a) $\lim_{x\to -3^-} f(x) = -\infty$

(b) $\lim_{x\to -3^+} f(x) = \infty$

(c) Graph $f(x)$